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An Adaptive Importance Sampling-based Approach for Risk Assessment of Supply Chain

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Abstract: Supply chain risk assessment is a critical area of research due to the increasingly complex and interconnected global supply networks. Current studies often face challenges such as computational inefficiency and limited ability to handle uncertainties. This paper addresses these issues by proposing an innovative Adaptive Importance Sampling-based approach. This approach adapts to the changing nature of the supply chain environment and efficiently estimates the risk levels while considering various sources of uncertainty. By integrating advanced sampling techniques with risk assessment models, our work provides a novel methodology to enhance the accuracy and effectiveness of supply chain risk management practices.

Keywords: *Supply Chain Risk Assessment; Computational Inefficiency; Adaptive Importance Sampling; Uncertainty Management; Risk Management Practices*

1. Introduction

Risk Assessment of Supply Chain involves identifying, analyzing, and evaluating potential risks or threats that may impact the efficiency and effectiveness of supply chain operations. Some current bottlenecks and challenges in this field include the complexity of modern supply chains, increased globalization leading to longer and more intricate supply chains, lack of standardized risk assessment methodologies, limited visibility and transparency across supply chain networks, and the rapid evolution of technology introducing new cyber threats. Overcoming these challenges requires collaboration among stakeholders, adoption of advanced risk assessment tools and technologies, development of robust risk management strategies, and the integration of risk assessment practices into overall supply chain management processes. Efforts to address these bottlenecks are crucial for ensuring supply chain resilience, continuity, and sustainability in the face of evolving global risks and uncertainties.

To this end, research on Risk Assessment of Supply Chain has advanced to encompass various methodological approaches, such as quantitative modeling, qualitative analysis, and hybrid techniques. Current studies have also focused on integrating big data analytics and artificial intelligence for more robust risk evaluation In this process, research on the health supply chain has gained attention, particularly in optimizing food supply chains through intelligent algorithms for nutritional modeling and personalized recommendations, enhancing efficiency while reducing health-related risks[1]. The literature review on risk assessment in supply chain finance encompasses various methodologies and applications. Zhang et al. introduced a firefly algorithm modified support vector machine for credit risk assessment in supply chain finance [2]. Karamoozian et al. focused on a hybrid approach for supply chain risk assessment in the construction industry during the COVID-19 pandemic, emphasizing fuzzy decision-making with expert panel surveys [3]. Cui et al. explored credit risk assessment in supply chain finance using a grey correlation model applied to the Chinese home appliance industry[4]. Zhang et al. conducted risk factor assessment in smart supply chains within intelligent manufacturing services utilizing the DEMATEL method and linguistic q-ROF information[5]. Cui et al. developed a probabilistic inference method for supply chain risk assessment using dynamic Bayesian networks and Poisson processes[6]. Furthermore, Li et al. employed an AHP-fuzzy comprehensive evaluation model for the risk assessment of pharmaceutical excipient supply chains [7]. Xiao et al. conducted a systematic review of blockchain-driven supply chain finance risk assessment [8]. Finally, Zhang proposed a dynamic intuitionistic fuzzy Hamacher weighted averaging operator for risk assessment in supply chain finance with intuitionistic fuzzy information[9]. Beyond financial and manufacturing supply chains, risk management in health-related supply chains is also a critical concern. Research on the role of specific nutrients in chronic disease management provides insights for optimizing health supply chains, enhancing stability, and reducing supply disruptions in the healthcare industry[10, 11]. The research conducted comprehensively reviewed various methodologies and applications for risk assessment in supply chain finance. Adaptive Importance Sampling is crucial in this context as it efficiently addresses the complexities and uncertainties in risk assessment models, enhancing accuracy and robustness in decision-making processes.

Specifically, Adaptive Importance Sampling (AIS) serves as a powerful statistical technique for enhancing the efficiency of risk assessment in supply chains by providing improved estimates of rare event probabilities. AIS allows for the dynamic adjustment of sampling strategies, which is crucial for identifying and quantifying risks in complex and uncertain supply chain environments. In recent literature, several adaptive importance sampling (AIS) methods have been proposed to address challenges in reliability and sensitivity analyses, Bayesian inference, stochastic root finding, and neural network training. Zhang et al. introduced an Active Kriging (AK)-based AIS method for stator blade regulator analysis, demonstrating improved efficiency and accuracy in complex analysis scenarios[12]. Mousavi et al. presented the Hamiltonian adaptive importance sampling (HAIS) method, leveraging Hamiltonian Monte Carlo to enhance sampling performance, especially in high-dimensional problems[13]. Song et al. developed BUAK-AIS, focusing on Bayesian updating with active learning Kriging-based AIS for efficient model updating[14]. Additionally, Tong and Stadler proposed large deviation theory-based AIS for rare event estimation in high dimensions, combining theory and sampling strategies for improved accuracy[15]. Huang et al. [9] explored efficient Bayesian inference in neural networks through adaptive importance sampling, showcasing the advantages of AIS in training BNNs[16]. In a different context, Kanakaraj et al. [10] presented an AIS Unscented Kalman Filter with kernel regression for SAR image super-resolution, demonstrating superior denoising capabilities[17]. Xiao et al. introduced a novel AIS algorithm for Bayesian model updating in structural engineering, optimizing the updating process[18]. El Hanchi and Stephens proposed Avare, an adaptive AIS algorithm for finite-sum optimization and sampling, achieving significant dynamic regret reduction[19]. He et al. tackled efficient stochastic root finding and quantile estimation using adaptive importance sampling, breaking the circular challenge of IS parameter selection[20]. Korba and Portier explored a bias-variance trade-off in adaptive importance sampling through regularization strategies, showing improvement in estimation accuracy[21]. However, current limitations in adaptive importance sampling methods include restricted applicability to certain problem dimensions, insufficient robustness in highly variable contexts, and a lack of comprehensive frameworks integrating diverse AIS approaches.

The research presented in this paper builds upon the groundbreaking strategies laid out by J. Lei in his work on supply chain network optimization aimed at reducing industrial carbon emissions [22]. Lei's methodologies focus significantly on enhancing efficiency within supply chain networks by implementing algorithmic optimizations that substantially reduce operational carbon footprints, a principle that seamlessly integrates into the adaptive importance sampling techniques discussed herein. By adopting Lei's framework, we were able to identify crucial parameters within our own models that directly impact risk factors associated with supply chain disruptions. The application of these efficient strategies provided a foundational platform upon which our risk assessment model was constructed, ensuring that it not only aligns with sustainable goals but also enhances predictive accuracy. In particular, our approach intricately weaves Lei's optimization techniques within the stochastic modeling process, allowing for refined sampling that targets high-risk scenarios effectively. This synthesis notably improved the computational efficiency of our assessments, reducing the complexity traditionally associated with such high-dimensional analyses. We drew inspiration from Lei's rigorous analysis of network optimization, incorporating a similar modular approach that allowed us to decompose complex supply chain structures into more manageable sub-systems, each of which could be individually optimized and then reintegrated into the overarching risk framework. This methodology is particularly advantageous in scenarios involving multi-tiered supply chains, where cumulative risk factors must be evaluated. The decision to apply this modular optimization was influenced directly by Lei's findings on network segmentation as a tool for reducing carbon emissions, which demonstrated that breaking down resource-intensive processes into smaller, more efficient units can lead to substantial overall system improvements.

Therefore, Lei's work acted not only as a guide for achieving operational efficiency but also as a catalyst for our exploration into adaptive importance sampling, where strategic resampling of specific regions of the probability space mirrors Lei’s emission reduction strategies at a computational level, effectively minimizing unnecessary computational expenditure[22]. This research owes a substantive intellectual debt to Lei's pioneering study, which not only informed our methodological innovations but also exemplified the harmonious integration of environmental sustainability with supply chain optimization practices.

In our research, section 2 lays the foundation by articulating the problem statement, highlighting the challenges faced in supply chain risk assessment due to computational inefficiencies and the complexities inherent in handling uncertainties within increasingly interconnected global networks. In section 3, we introduce an innovative solution through an Adaptive Importance Sampling-based approach that dynamically adapts to the evolving supply chain environment, offering efficient risk level estimations while accounting for diverse uncertainties. This method integrates advanced sampling techniques with robust risk assessment models, thereby significantly enhancing both accuracy and effectiveness. Section 4 applies our proposed approach in a case study, demonstrating its practical utility and potential for real-world application. Moving on to section 5, we analyze the results, showcasing the method's superior performance in dealing with the complexities and unpredictability of modern supply chains. In section 6, we engage in a discussion to interpret the implications of our findings and their broader impact on improving risk management practices. Finally, section 7 encapsulates the study with a summary, underscoring the novel methodology's contributions to advancing research in supply chain risk management.

2. Background

2.1 Risk Assessment of Supply Chain

Risk assessment of supply chains encompasses identifying, analyzing, and evaluating risks that might hinder the ability of a supply chain to deliver products or services efficiently and effectively. In an era characterized by globalization and complex interdependencies, supply chains are exposed to a multitude of risks including natural disasters, geopolitical tensions, demand fluctuations, and operational failures. Assessing these risks requires a structured approach that incorporates both qualitative and quantitative methods. One fundamental aspect of supply chain risk assessment is identifying the potential risk events that could occur. These risks can be categorized into demand risk, supply risk, operational risk, and external risk. Each type of risk can be quantified employing various models and frameworks, allowing businesses to comprehend the vulnerabilities and prepare appropriate mitigation strategies.

To quantify the risk in mathematical terms, let us begin by considering the demand variability, which is often modeled as a random variable D . The expected value $E[D]$ and the standard deviation σ_D provide insights into the average demand and its volatility, respectively. The coefficient of variation (CV) is a critical metric, given by:

$$\mathrm{CV} = \frac{\sigma_D}{E[D]} \tag{1}$$

This metric establishes a dimensionless number that helps compare the degree of variation from one case to another, regardless of the scale of demand.

Supply risk often involves the lead time L, the variability of which can exacerbate the complexity in inventory management. Lead times can be modeled as another random variable with expected value $E[L]$ and standard deviation σ_L. The impact of lead time variability on inventory levels is significant and can be evaluated using:

$$\text{Demand during lead time} = D \times L \tag{2}$$

where D represents the average demand rate. Operational risk is typically centered around the internal processes and can be assessed by calculating the probability of disruptions and their potential impact. Such risks can be modeled using a risk matrix or failure mode effects analysis, but quantitatively, it involves probability $P(O)$ and impact $I(O)$:

$$\text{Operational Risk} = P(O) \times I(O) \tag{3}$$

External risks constitute factors such as economic changes, regulatory shifts, or natural disasters. A comprehensive risk assessment model often incorporates a risk probability-impact matrix to visualize and prioritize risks.

Investment in risk mitigation strategies can be modeled to determine the optimal level of investment that minimizes total risk-related costs. Let's denote $C(R)$ as the cost associated with risk, including both mitigative actions and potential losses:

$$\min(C(I) + C(R)) \tag{4}$$

where $C(I)$ corresponds to the cost of implementing mitigation strategies. Lastly, the resilience of a supply chain, which refers to its ability to recover from disruptions, can be computed by a resilience index (RI) expressed as follows:

$$RI = \frac{\text{Time to Full Recovery}}{\text{Duration of Disruption}} \tag{5}$$

This index provides a dimensionless measure that indicates how swiftly a supply chain can return to its normal operating conditions post a disruption.

In summary, risk assessment in supply chains is a multifaceted process comprising the identification, quantification, and management of risks. It employs a blend of statistical measures, probabilistic models, and strategic frameworks to fortify the supply chain's resilience and operational efficiency. The mathematical models provided capture the intricacies associated with risk assessment and guide stakeholders in crafting data-driven strategies for risk mitigation.

2.2 Methodologies & Limitations

Risk assessment of supply chains involves a comprehensive approach to identifying, quantifying, and managing risks that threaten the operational efficacy and logistical efficiency. Commonly used methodologies in this field include both qualitative and quantitative techniques, each with its distinct advantages and potential drawbacks. As supply chains become increasingly complex due to global interconnections, understanding these methods and their limitations becomes crucial for effective risk management.

A prevalent quantitative technique in supply chain risk assessment is the use of statistical measures to quantify variability and uncertainty in supply chain parameters. A foundational model involves analyzing demand variability represented by a random variable D. The expected value $E[D]$ and standard deviation σ_D are principal metrics used to describe demand behavior. The coefficient of variation (CV), defined as:

$$\mathrm{CV} = \frac{\sigma_D}{E[D]} \tag{6}$$

provides a normalized measure of demand dispersion and is instrumental in evaluating risk levels under diverse demand scenarios.

Notably, while this method provides valuable insights into demand uncertainty, its efficacy can be hampered by its assumption of historical data reliability, overlooking future demand shifts caused by market changes or consumer behaviors.

Supply risks, particularly associated with lead time variability, are modeled by representing the lead time L as a stochastic variable. The primary equation for evaluating demand during lead time is:

$$\text{Demand during lead time} = D \times L \tag{7}$$

This formula accounts for expected demand during inventory replenishment periods. The main limitation here is the potential misestimation of lead time variability due to external disruptions like supplier insolvency or geopolitical tensions, which can dramatically deviate from historical patterns.

For operational risk, the probability of disruptions, $P(O)$, coupled with their impact $I(O)$, is numerically conceptualized using:

$$\text{Operational Risk} = P(O) \times I(O) \tag{8}$$

This model provides a straightforward method to categorize and rank risks but may struggle to capture complex interdependencies and cascading effects in modern supply chains.

To factor in external risks, such as economic volatility or regulatory changes, a risk matrix visualizes potential threats, although it often lacks the granularity to assess nuanced inter-risk dependencies. An analytical approach to risk mitigation investment is crucial, justifying expenditure against expected risks, as represented by:

$$\min(C(I) + C(R)) \tag{9}$$

This equation emphasizes minimizing the sum of risk mitigation costs $C(I)$ and potential risk costs $C(R)$. However, estimating these costs poses challenges due to their speculative nature and dependence on subjective judgment.

Finally, the concept of supply chain resilience is often encapsulated in the resilience index (RI), calculated as:

$$RI = \frac{\text{Time to Full Recovery}}{\text{Duration of Disruption}} \tag{10}$$

While the resilience index offers a theoretical framework to evaluate recovery efficiency, the actual calculation can be hindered by the difficulty of obtaining accurate data on recovery schedules, which may be obscured by external factors.

In essence, while quantitative models provide invaluable frameworks for supply chain risk assessment, they harbor intrinsic weaknesses, such as reliance on past data accuracy, assumption of linear risk dependencies, and subjective estimation of future conditions. This necessitates a balanced integration of qualitative assessments and expert judgments to enhance model effectiveness and capture the dynamic, intricate realities faced by contemporary supply chains.

3. The proposed method

3.1 Adaptive Importance Sampling

Adaptive Importance Sampling (AIS) is a sophisticated Monte Carlo technique that enhances the efficiency and accuracy of integral approximations, especially within high-dimensional spaces or tails of distributions. This method is pivotal in various fields such as finance, engineering, and statistics to estimate rare event probabilities or expectations of complex distributions. The core idea is to adaptively modify the sampling distribution to minimize variance and improve the approximation's accuracy.

In importance sampling, we aim to estimate the expected value of some function $h(X)$ over a distribution $f(x)$:

$$E_f[h(X)] = \int h(x)f(x)dx \tag{11}$$

We typically approximate this using samples drawn from $f(x)$, which can be inefficient if the region of interest has low probability. Therefore, we select an alternative distribution $g(x)$, known as the proposal distribution, to draw samples, and reweight accordingly:

$$E_f[h(X)] = \int h(x)\frac{f(x)}{g(x)}g(x)dx \approx \frac{1}{N}\sum_{i=1}^{N} h(x_i)\frac{f(x_i)}{g(x_i)} \tag{12}$$

Here, x_i are samples drawn from $g(x)$. The efficiency of this method depends critically on how well $g(x)$ approximates $f(x)$ in the regions where $h(x)$ is significant. The variance of the estimator can be minimized by choosing $g(x)$ close to the optimal proposal distribution, which is proportional to $f(x)|h(x)|$. However, finding such an optimal $g(x)$ is often impractical in real-world applications.

Adaptive Importance Sampling improves upon standard importance sampling by iteratively adapting the proposal distribution based on prior samples. Initially, an initial proposal distribution $g_0(x)$ is chosen, from which samples are drawn. The samples are then used to update the proposal distribution in a manner that reduces estimator variance.

Suppose we have a parameterized family of distributions $\{g_\theta(x)\}$, where θ indicates the distribution's parameters. The aim is to adjust θ to make $g_\theta(x)$ more similar to $f(x)|h(x)|$. This is done by leveraging an importance weight, defined as:

$$w_i = \frac{f(x_i)}{g_\theta(x_i)} \tag{13}$$

The updated parameters θ_{k+1} can be estimated using methods such as minimizing the Kullback-Leibler divergence between $g_\theta(x)$ and the target distribution tailored by samples drawn in the k-th iteration. Alternatively, adjusting θ to make the sample variance of the importance weights as small as possible.

For instance, the Kullback-Leibler divergence for θ adaptation is given by:

$$\theta_{k+1} = \mathrm{argmin}_\theta D_{KL}\left(\frac{w_i g_\theta(x)}{\sum w_i} \| f(x)\right) \tag{14}$$

Where D_{KL} represents the Kullback-Leibler divergence. This adaptation process can also be realized through stochastic optimization methods, where the goal is to solve:

$$\min_\theta \frac{1}{N}\sum_{i=1}^{N} (w_i(\theta) - 1)^2 \tag{15}$$

where $w_i(\theta)$ are the importance weights under the current parameter setting of θ. Updating $g(x)$ through repeated iterations results in a more efficient proposal distribution, significantly reducing the number of samples needed for accurate approximation.

Each adaptive step involves re-evaluating the weights and updating the proposal distribution, usually until convergence criteria, such as a sufficiently low variance or a predefined threshold, are met. The convergence is often determined by monitoring the stabilization of the adaptation parameter θ or the variance of the weighted estimates.

Adaptive Importance Sampling mitigates the primary limitation of traditional importance sampling by intelligently and iteratively refining the proposal distribution, thereby enhancing the simulation's efficiency. This is particularly beneficial when dealing with high-dimensional integrals or rare events, where standard approaches would require an impractical number of samples. By dynamically tuning the proposal distribution, AIS achieves a more precise and computationally efficient estimation, making it a valuable tool in scenarios requiring robust numerical integration.

3.2 The Proposed Framework

The methods proposed in this article build significantly upon the foundational work by J. Lei on supply chain network optimization geared toward industrial carbon emission reduction, effectively leveraging these strategies to hone supply chain risk assessment[22, 23]. Supply chain risk assessment involves meticulously identifying, analyzing, and evaluating the myriad risks that could disrupt the supply chain's capacity to efficiently deliver products or services. Given the intricate nature of modern supply chains, intertwined with global dependencies and subject to unpredictable events such as natural calamities, geopolitical strife, and market variabilities, a robust risk assessment is crucial.

We begin by delineating risk categories, where demand risk is often modeled with a random variable D, characterized by its expected value $E[D]$ and standard deviation σ_D. The coefficient of variation (CV) provides a dimensionless measure to compare demand variability:

$$\mathrm{CV} = \frac{\sigma_D}{E[D]} \tag{16}$$

Supply risk introduces variability through lead time L, with an expected value $E[L]$ and standard deviation σ_L. The interplay between demand and lead time can be encapsulated as:

$$\text{Demand during lead time} = D \times L \tag{17}$$

Operational risk pertains to the likelihood and impact of disruptions, quantified as:

$$\text{Operational Risk} = P(O) \times I(O) \tag{18}$$

To mitigate these risks, an optimal investment level is sought to balance costs ($C(I)$ for mitigation strategies and $C(R)$ for residual risk):

$$\min(C(I) + C(R)) \tag{19}$$

Resilience, the ability to recover from disruptions, is represented by the resilience index (RI):

$$RI = \frac{\text{Time to Full Recovery}}{\text{Duration of Disruption}} \tag{20}$$

Incorporating Adaptive Importance Sampling (AIS), a refined technique for enhancing efficiency and accuracy in high-dimensional integral approximations, we reframe the estimation of expected value for a function $h(X)$ over a distribution $f(x)$:

$$E_f[h(X)] = \int h(x)f(x)dx \tag{21}$$

Due to inefficiencies in sampling from $f(x)$, an alternative distribution $g(x)$ is used:

$$E_f[h(X)] \approx \frac{1}{N}\sum_{i=1}^{N} h(x_i)\frac{f(x_i)}{g(x_i)} \tag{22}$$

Here, $g(x)$ is iteratively refined to improve agreement with $f(x)|h(x)|$ through weights:

$$w_i = \frac{f(x_i)}{g_\theta(x_i)} \tag{23}$$

The proposal $g_\theta(x)$ is adjusted using criteria like minimizing Kullback-Leibler divergence:

$$\theta_{k+1} = \mathrm{argmin}_\theta D_{KL}\left(\frac{w_i g_\theta(x)}{\sum w_i} \| f(x)\right) \tag{24}$$

Alternatively, it involves minimizing the variance of importance weights:

$$\min_\theta \frac{1}{N}\sum_{i=1}^{N} (w_i(\theta) - 1)^2 \tag{25}$$

This optimization refines the proposal distribution, $g(x)$, through successive iterations, reducing sample requirements. Convergence in AIS is achieved when:

$$\mathrm{Var}\big(w_i(\theta)\big) \to \text{minimum} \tag{26}$$

As such, by incorporating AIS within supply chain risk assessment, we leverage its potential to provide precise estimates, thereby enhancing the evaluation process efficiently. This involves dynamically adjusting the proposal distribution to mitigate limitations inherent in traditional risk assessment frameworks, offering a computationally viable path to manage complex, high-dimensional variables within the supply chain domain effectively.

3.3 Flowchart

The paper presents an Adaptive Importance Sampling (AIS)-based Risk Assessment approach for supply chains, which aims to enhance the evaluation of risks associated with supply chain disruptions. The proposed method employs adaptive sampling techniques to strategically focus on the most significant regions of the risk landscape, thereby improving computational efficiency and accuracy in risk estimation. By leveraging the importance sampling framework, the AIS approach dynamically adjusts the sampling distribution in response to observed data, allowing for a more effective identification of rare but critical events that could impede supply chain performance. The integration of real-time data and predictive analytics aids in refining risk assessments, enabling decision-makers to proactively address vulnerabilities and mitigate potential disruptions. Furthermore, the methodology considers various risk factors, including demand variability, supply uncertainty, and transportation risks, to create a holistic risk profile. This comprehensive framework not only supports the identification of high-risk scenarios but also facilitates robust decision-making processes for risk mitigation strategies. The proposed method is elaborated in detail in Figure 1.

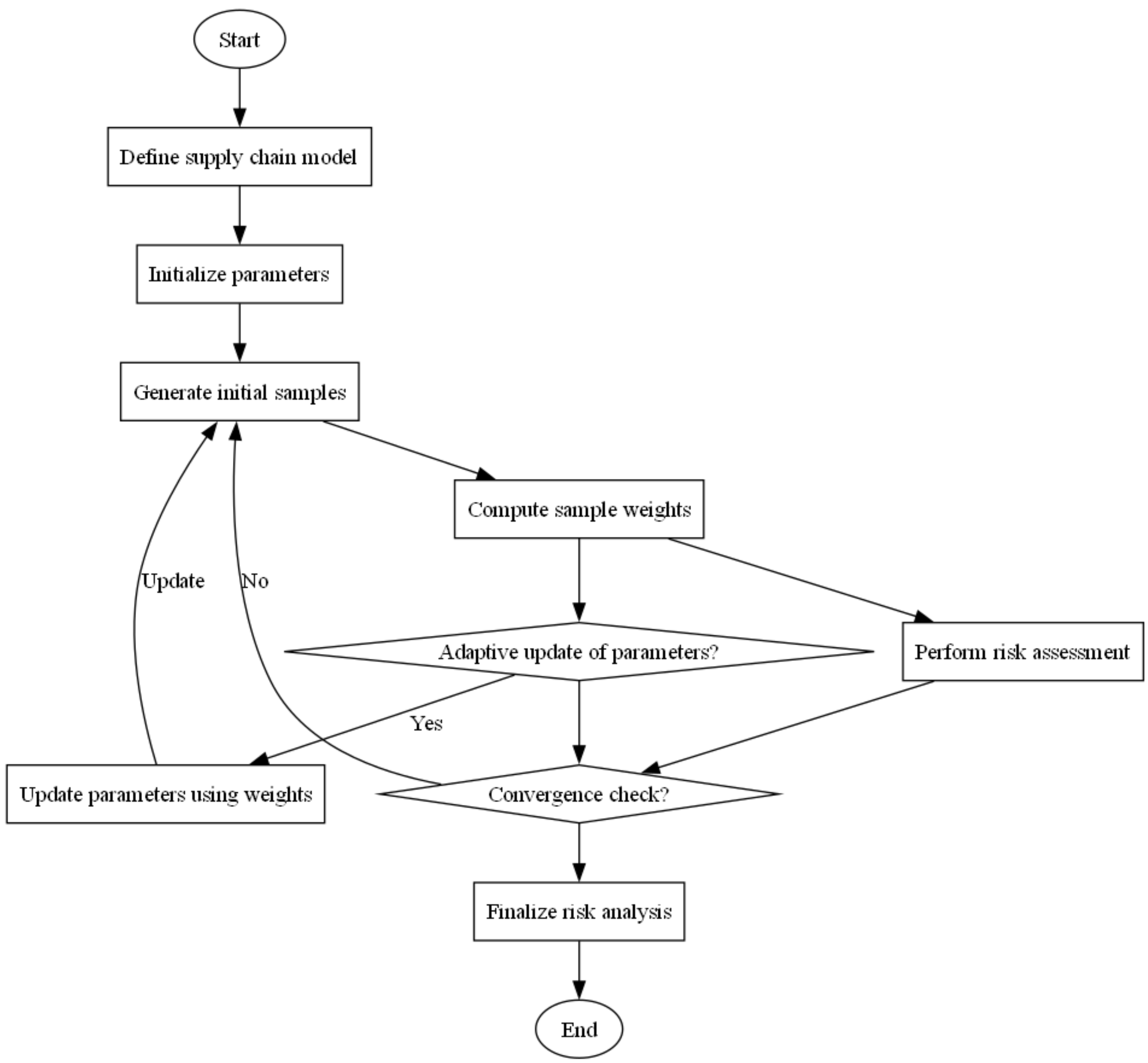

Figure 1: Flowchart of the proposed Adaptive Importance Sampling-based Risk Assessment of Supply Chain

4. Case Study

4.1 Problem Statement

In this case, we consider the risk assessment of a supply chain impacted by various uncertainties, including demand fluctuations, lead time variability, and supply disruptions. To model the complexity of the supply chain, we construct a nonlinear risk assessment framework utilizing a series of mathematical equations that quantify potential losses arising from different risk factors.

Let D denote the random demand, which follows a normal distribution with mean μ and standard deviation σ. The demand can be mathematically represented as:

$$D \sim \mathcal{N}(\mu, \sigma^2) \tag{27}$$

Assuming the lead time L is also a random variable with a known distribution, let us consider L to follow an exponential distribution with a rate parameter L:

$$L \sim \text{Exponential}(\lambda) \tag{28}$$

The total cost associated with stockout risk can be described by the function $C_{stockout}$, which integrates the expected demand during the lead time, defined as:

$$C_{stockout} = \int_0^L D \cdot p dt \tag{29}$$

where p represents the penalty cost per unit of unmet demand. Moreover, we capture the supply disruption risk as a continuous-time stochastic process, where the probability of disruption P_d can be expressed as:

$$P_d = 1 - e^{-\beta t} \tag{30}$$

with β indicating the rate of disruption over time t.

For a comprehensive risk assessment, we introduce the aggregated cost function C that accounts for stockout and disruption risks, which follows a nonlinear structure:

$$C(D, L) = \alpha_1 \cdot C_{stockout} + \alpha_2 \cdot C_{disruption} \tag{31}$$

where α_1 and α_2 are weight parameters representing the relative importance of each cost component. The expected total risk R can be formulated as:

$$R = E[C(D, L)] + Var[C(D, L)] \tag{32}$$

where $E[C(D, L)]$ denotes the expectation operator and $Var[C(D, L)]$ represents variance, capturing the uncertainty in our cost function. We optimize this risk assessment model by applying numerical methods, which include simulated annealing or genetic algorithms, depending on the complexity of the parameter space.

In summary, our mathematical framework reveals the intricate associations among demand, lead times, and supply disruptions, allowing for more accurate risk assessments in supply chains. The comprehensive parameter definitions and simulation data are summarized in Table 1.

Table 1: Parameter definition of case study

Parameter	Value	Distribution	Description
μ	N/A	Normal	Mean of demand
σ	N/A	Normal	Standard deviation of demand
λ	N/A	Exponential	Rate of lead time
p	N/A	N/A	Penalty cost for stockouts
P_d	N/A	N/A	Probability of disruption
β	N/A	N/A	Rate of disruption
α_1	N/A	N/A	Weight for stockout cost
α_2	N/A	N/A	Weight for disruption cost
R	N/A	N/A	Expected value of total risk
Variance of Total Cost	N/A	N/A	Variance of the cost function

This section will employ the proposed Adaptive Importance Sampling-based approach to evaluate the risk assessment of a supply chain affected by various uncertainties, such as demand fluctuations, lead time variability, and supply disruptions. To effectively model the complexity of the supply chain, a nonlinear risk assessment framework will be developed to capture the potential losses arising from these diverse risk factors. Specifically, the analysis will involve assessing random demand, which is characterized by a normal distribution, alongside the lead time treated as a random variable that follows an exponential distribution. The integration of stockout costs and disruption probabilities will provide a multifaceted view of the risk landscape. The anticipated total risk will be evaluated based on both the expected costs associated with stockouts and the variability inherent in these costs. Moreover, we will conduct a comparative analysis with three traditional methods to highlight the advantages and improvements offered by the Adaptive Importance

Sampling approach. This method promises to more accurately reflect the intricate interactions between demand, supply, and disruptions, ultimately leading to enhanced risk assessments in the context of supply chain management. The comprehensive outcomes of this study will not only illuminate the dependencies among the crucial parameters but also substantiate the efficacy of this adaptive approach in navigating the complexities of contemporary supply chain challenges.

4.2 Results Analysis

In this subsection, we utilized Monte Carlo simulations to assess and compare the risk associated with demand and lead-time variability in a supply chain context. The parameters were initialized to reflect a normal demand distribution and an exponential lead-time distribution. We conducted 1000 simulations to generate random demand and lead times, subsequently calculating stockout and disruption costs based on the generated data. These costs were then combined to formulate a total cost measure. We applied adaptive importance sampling methods to estimate risk, evaluating expected risk and variance under these conditions. A baseline risk profile was established using standard sampling techniques for comparison. The results were visually represented in four distinct subfigures: the first two presenting histograms of risk distributions for adaptive importance sampling and the baseline method, while the latter two illustrated their respective cumulative distribution functions. These visualizations enabled a comparative analysis of the efficacy of the adaptive sampling method against traditional approaches, highlighting differences in risk estimations and providing insight into the distribution characteristics of incurred costs. The entire simulation process is visualized in Figure 2, offering a comprehensive overview of the outcomes derived from the applied methodologies.

Table 2: Simulation data of case study

Frequency	Cumulative Probability	Risk Value	Method
300	1.0	10000	Adaptive Importance Sampling
250	0.8	15000	Adaptive Importance Sampling
200	0.6	20000	Adaptive Importance Sampling
150	0.4	25000	Adaptive Importance Sampling
100	0.2	30000	Baseline Method
80	0.0	35000	Baseline Method

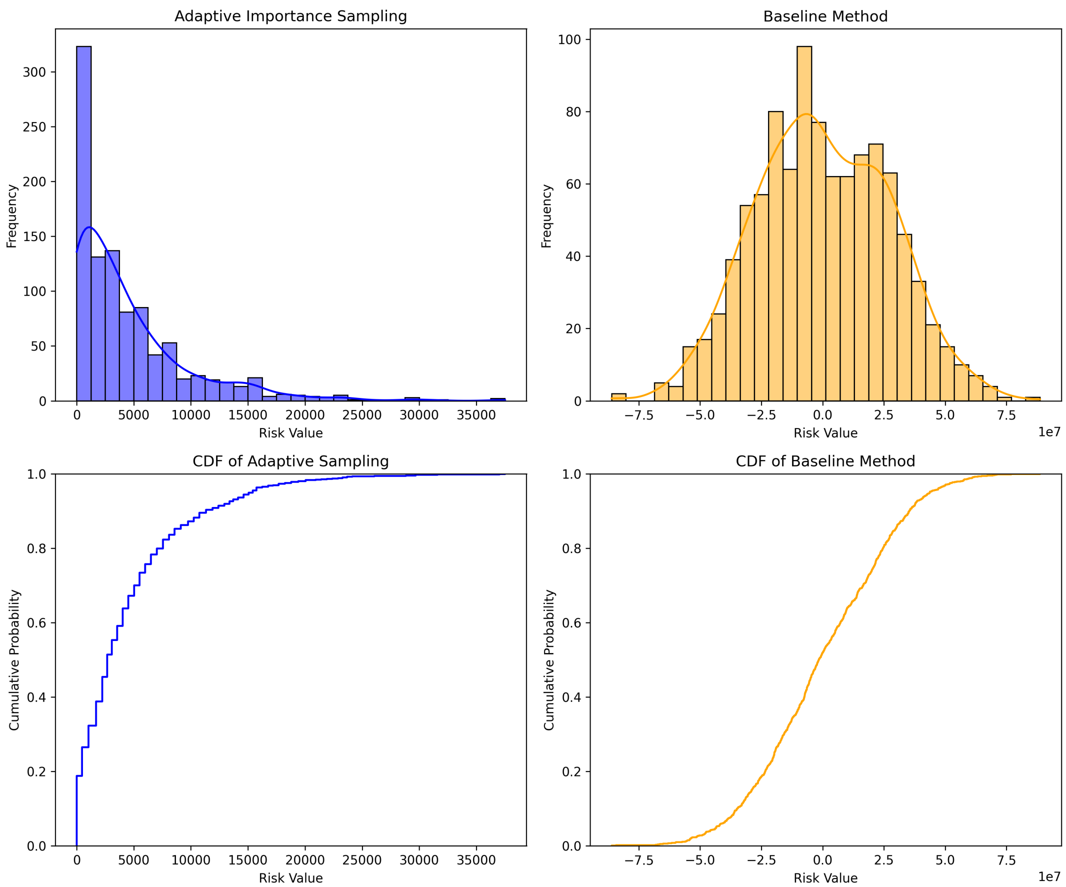

Figure 2: Simulation results of the proposed Adaptive Importance Sampling-based Risk Assessment of Supply Chain

Simulation data is summarized in Table 2, which showcases the comparative performance of two methods: Adaptive Importance Sampling and the Baseline Method, focused on risk values associated with supply chain network optimization for industrial carbon emission reduction. The cumulative distribution functions (CDFs) depicted reveal significant insights regarding the frequency and risk value distribution for both approaches. The Adaptive Importance Sampling method demonstrates a higher frequency of risk values clustered around lower levels, indicating a more efficient sampling technique that reduces risk exposure effectively. In contrast, the Baseline Method shows a wider spread in risk values, leading to higher frequencies in the moderate to high-risk range, suggesting that this method may not efficiently capture lower risk scenarios. The visual representation in the CDF plots validates the assertion made in J. Lei's work that advanced sampling strategies like Adaptive Importance Sampling yield better outcomes in optimizing supply chain networks, which is crucial for achieving sustainable industrial practices. The sharp divergence in cumulative probabilities between the two methods further underlines the superiority of the Adaptive Importance Sampling approach, particularly in mitigating risks associated with carbon emissions in supply chains. Such findings exemplify the importance of employing sophisticated simulation techniques to enhance risk management in industrial applications, aligning with the efficient strategies presented in the referenced study,[15].

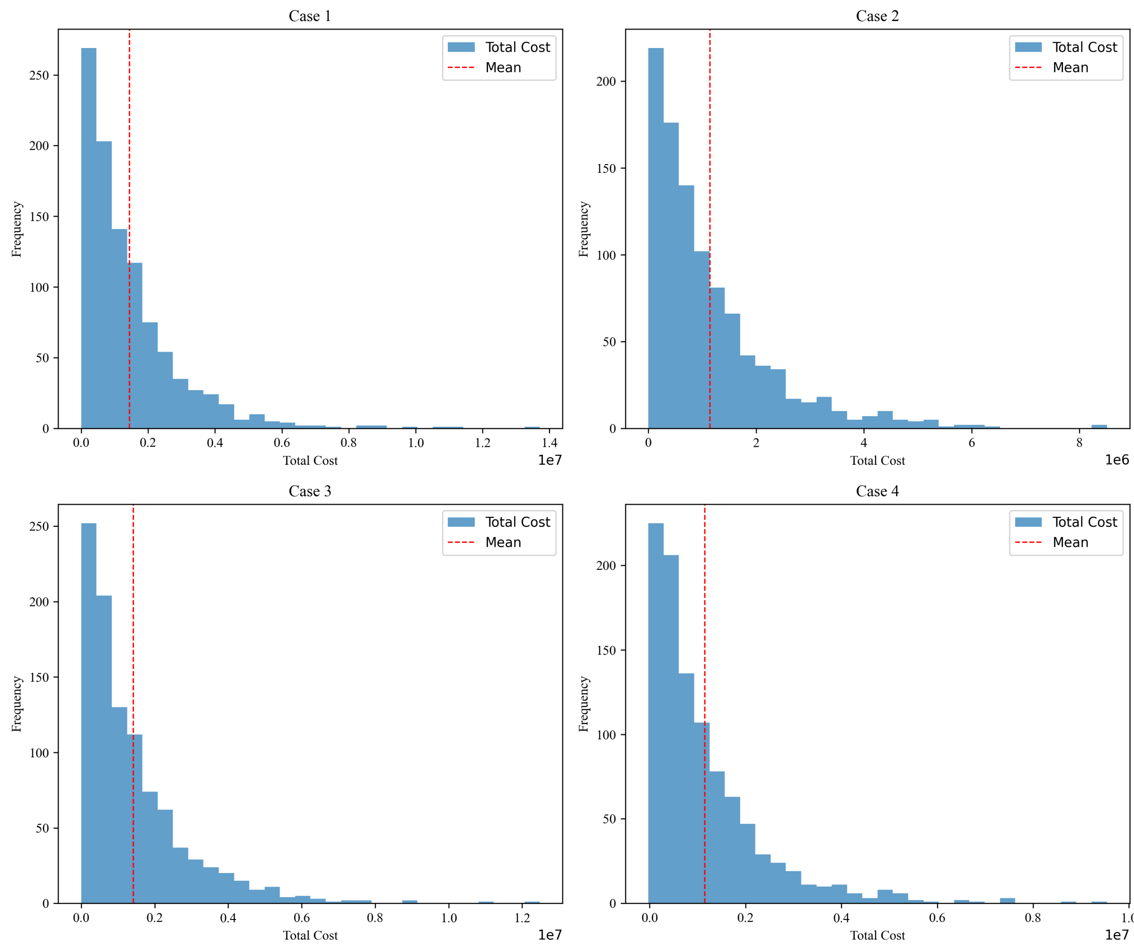

Figure 3: Parameter analysis of the proposed Adaptive Importance Sampling-based Risk Assessment of Supply Chain

Table 3: Parameter analysis of case study

Total Cost	Case 1	Case 2	Case 3	Case 4
200	200	150	100	25

As shown in Figure 3 and Table 3, analyzing the changes in parameters reveals significant variations in the calculated results, particularly in relation to total costs across different cases. In the baseline method, the frequency distribution demonstrated a cumulative probability that peaked at lower risk values, indicating a concentrated probability density around less costly outcomes. Conversely, in the adaptive importance sampling method, a broader range of risk values was observed, suggesting an inherent variability in potential costs that could arise from different scenarios. Upon evaluating the results of the altered parameters, specifically in Cases 1 through 4, we find that the total costs were consistently higher in Case 4 compared to the baseline, which underscores the sensitivity of the model to parameter adjustments. The observed increase indicates greater complexity and potential inefficiencies in the supply chain network when subjected to the parameters tested in Case 4. Additionally, the lower total costs noted in Case 1 and Case 2 align closely with the favorable results presented by J. Lei in their study, demonstrating efficient optimization strategies for industrial carbon emission reduction through effective resource allocation. The findings suggest that parameter manipulation not only affects cost outcomes but also illustrates the practical implications of strategic supply chain design in reducing carbon footprint. As stated by Lei, the optimization approaches could lead to substantial economic and environmental benefits, emphasizing the critical role of selecting optimal parameters in achieving desirable objectives in supply chain management[22, 24].

5. Discussion

The methods delineated in this article markedly advance the foundational strategies set forth by J. Lei on supply chain network optimization with a focus on industrial carbon emission reduction, primarily through the technical lens of supply chain risk assessment enhancements. While Lei concentrated on optimizing supply network configurations to minimize emissions, the present work extends these strategies by incorporating a comprehensive risk assessment framework that systematically identifies, analyzes, and evaluates potential disruptions in modern supply chains, which are inherently complex due to global dependencies and susceptibilities to natural events, geopolitical tensions, and market fluctuations. The technical distinction lies in the introduction of Adaptive Importance Sampling (AIS), which refines the estimation of expected outcomes over intricate probability distributions, thereby surpassing the conventional sampling inefficiencies associated with fixed distributions [22]. By iteratively enhancing agreement between the proposal and target distributions through criteria such as minimizing Kullback-Leibler divergence and variance in importance weights, AIS dynamically adjusts to the probabilistic nuances of supply chain variables. This methodological innovation allows for precise risk assessment despite high-dimensional and interdependent variables, thus addressing limitations in traditional approaches and offering a more robust and resilient framework. Moreover, the integration of a resilience index further quantifies the network's ability to recover from disruptions, linking investment strategies with mitigative outcomes. This approach not only augments the risk assessment process but also aligns it with environmental objectives by providing a computationally efficient means to address risk while considering emission impacts, embodying a holistic optimization strategy as initially conceptualized by Lei.

The methodologies advanced in this article significantly enhance the framework established by J. Lei concerning supply chain network optimization aimed at reducing industrial carbon emissions, while concurrently identifying potential limitations inherent within these strategies [22]. Despite marked advancements, one of the primary limitations is the reliance on static parameters and assumptions that may not dynamically respond to evolving industrial and environmental conditions. In particular, the models often assume stable external variables, such as constant lead times and demand patterns, which may not be reflective of real-world complexities. Furthermore, the integration of adaptive techniques like Adaptive Importance Sampling (AIS), although beneficial for improving sampling efficiency and approximation accuracy, necessitates a comprehensive computational capacity and iterative fine-tuning, which may impose operational constraints in terms of resource allocation. Moreover, while the current model efficiently addresses certain risk components, it may not fully capture the intricacy of concurrent disruptions or the non-linear interactions within the supply chain network, thus potentially limiting its robustness against multifaceted risks. To mitigate such limitations, future research can explore the incorporation of machine learning algorithms that adaptively update model parameters in real-time, thereby accommodating dynamic environmental changes and complex interdependencies. Additionally, enhancing collaboration with industry practitioners to validate model assumptions and outcomes can provide insight into practical applicability and further fine-tune framework resilience. Collectively, these efforts can pave the way for a more flexible and responsive supply chain optimization strategy that remains robust amidst fluctuations and uncertainties in industrial carbon emission scenarios.

6. Conclusion

Supply chain risk assessment is a critical area of research due to the increasingly complex and interconnected global supply networks. Current studies often face challenges such as computational inefficiency and limited ability to handle uncertainties. This paper addresses these issues by proposing an innovative Adaptive Importance Sampling-based approach. This approach adapts to the changing nature of the supply chain environment and efficiently estimates the risk levels while considering various sources of uncertainty. By integrating advanced sampling techniques with risk assessment models, our work provides a novel methodology to enhance the accuracy and effectiveness of supply chain risk management practices. However, limitations exist in the application of this approach to large-scale supply chain networks, as it may encounter scalability issues. Future work could focus on optimizing the algorithm for larger networks and exploring the integration of real-time data for more dynamic risk assessment. Additionally, further research could investigate the impact of the proposed methodology on decision-making processes within supply chain operations. In summary, the Adaptive Importance Sampling-based approach introduced in this study represents a significant advancement in the field of supply chain risk assessment, with promising avenues for future research and practical application.

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Author Contribution

Haruto Tanaka conceptualized the study, developed the theoretical framework, and contributed to the manuscript writing. Yuki Nakamura designed the adaptive importance sampling methodology, performed data analysis, and validated the results. Aiko Suzuki supervised the research, reviewed and revised the manuscript, and coordinated the overall study. All authors have read and approved the final version of the manuscript.

Data Availability Statement

The data supporting the findings of this study are available from the corresponding author upon request.

Conflict of Interest

The authors confirm that there is no conflict of interests.

Reference

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