



A Numerical Study on Supply Chain Optimization with Dynamic Bayesian Networks

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Abstract: This paper presents a numerical study on supply chain optimization using Dynamic Bayesian Networks. The optimization of supply chains is crucial in today's complex and dynamic business environment. However, existing research faces challenges in effectively modeling and optimizing supply chain processes due to their dynamic and uncertain nature. This study addresses this gap by introducing Dynamic Bayesian Networks as a novel approach to model the relationships and uncertainties in supply chain operations. The innovative aspect of this work lies in the development of a framework that integrates Dynamic Bayesian Networks with optimization algorithms to enhance supply chain performance. The findings of this research provide valuable insights for practitioners and researchers seeking to improve supply chain efficiency and resilience.

Keywords: *Supply Chain; Optimization; Dynamic Bayesian Networks; Framework Development; Performance Enhancement*

1. Introduction

Supply Chain Optimization is a field that focuses on improving the efficiency and effectiveness of the entire supply chain process, from raw material sourcing to the delivery of the final product to customers. However, there are several bottlenecks and challenges that researchers in this field currently face. These include the complexity of global supply chains, the need for real-time data and visibility, the impact of external factors such as natural disasters and geopolitical events, as

well as the implementation of advanced technologies like artificial intelligence and blockchain. Overcoming these challenges requires interdisciplinary collaboration, advanced analytics techniques, and a deep understanding of both the supply chain dynamics and the latest technological innovations. By addressing these obstacles, researchers aim to develop innovative solutions that can optimize supply chain operations, reduce costs, improve customer satisfaction, and drive business growth.

To this end, research on Supply Chain Optimization has advanced to encompass various methods such as mathematical modeling, artificial intelligence algorithms, and big data analytics. The integration of green transformation into supply chain management, particularly in the chemical industry, enhances environmental performance and significantly improves financial outcomes, highlighting the importance of sustainability in supply chain design[1, 2]. In recent years, supply chain optimization research has expanded, intersecting with health management, green transformation, and industrial clusters[3-5]. Current studies focus on addressing real-time decision-making, sustainability, and risk management challenges in supply chain operations, indicating a comprehensive and interdisciplinary approach to enhancing efficiency and competitiveness. The integration of Artificial Intelligence (AI) into supply chain management has emerged as a pivotal avenue for enhancing efficiency and resilience in contemporary business operations. The application value of the nutrition supply chain in health management highlights the potential strategic significance of supply chain optimization in meeting consumers' health needs[6]. Anber Abraheem Shlash Mohammad et al. investigate the benefits of AI-powered predictive analytics in improving competitiveness and effectiveness of supply chains, emphasizing the role of Supply Chain Optimization using Artificial Intelligence (SCO-AI) systems in enhancing logistics route optimization and real-time inventory control[7]. Nsiong Louis and Eyo-Udo provide a comprehensive review of AI integration in Supply Chain Management (SCM), noting significant advancements in AI technologies such as machine learning and their applications in demand forecasting and logistics optimization[8]. Meanwhile, Abaku et al. explore theoretical approaches to AI in supply chain optimization, highlighting machine learning for demand forecasting and inventory management, as well as the role of game theory and multi-agent systems in decision-making processes[9]. Adama et al. delve into the economic theory and practical impacts of digital transformation in supply chain optimization, addressing challenges such as cybersecurity threats and proposing strategic recommendations for AI adoption in SCM[10]. Joel et al. present a detailed review of current AI practices for supply chain optimization, emphasizing the transformative potential of AI-driven technologies across various supply chain processes[11]. Lastly, Zoubida Benmamoun et al. introduce the Wombat Optimization Algorithm (WOA) for supply chain optimization, demonstrating its effectiveness in managing exploration and exploitation to deliver optimal solutions for optimization problems[12]. Dynamic Bayesian Networks (DBN) are indispensable in AI integration within supply chain management due to their ability to model complex relationships, uncertainties, and temporal dependencies. Employing DBNs enables enhanced predictive analytics, optimized decision-making, and effective risk management, ultimately improving operational efficiency and resilience in supply chain operations.

Specifically, Dynamic Bayesian Networks (DBNs) provide a probabilistic framework to model the uncertainties and interdependencies in supply chains, enabling more effective decision-making. By capturing the dynamic relationships among supply chain components over time, DBNs enhance forecasting, risk assessment, and overall optimization strategies in supply chain management. Murphy and Russell extensively discuss the representation, inference, and learning aspects of DBNs, providing novel technical contributions such as representing Hierarchical HMMs as DBNs and introducing an exact smoothing algorithm[13]. Jafari et al. utilized DBNs for evaluating the resilience of engineering systems and reliability assessment of fire alarm systems, respectively[14]. Furthermore, Gomes and Wolf employed DBNs for health monitoring in autonomous vehicles, demonstrating the widespread applicability of DBNs in various domains[15]. In another study, Cai et al. proposed a DBN-based methodology for assessing the resilience of structure systems, specifically focusing on subsea oil and gas pipelines as a case study[16]. Moreover, Liu et al. developed a Fuzzy PLS-based DBN model for wastewater treatment processes, showcasing superior modeling performance compared to traditional approaches [17]. Luque and Štraub explored the application of DBNs for risk-based optimal inspection strategies in structural systems, highlighting the utility of DBNs in decision-making processes [18]. Lastly, Käser et al. introduced DBNs for student modeling, presenting enhanced predictive accuracy across different learning domains and offering insights into instructional policies [19]. However, current limitations of Dynamic Bayesian Networks (DBNs) include challenges in scalability for large datasets, computational intensity during inference, and difficulties in parameter estimation under complex conditions.

The methodologies explored in the study "Efficient Strategies on Supply Chain Network Optimization for Industrial Carbon Emission Reduction" by J. Lei have served as a substantial source of inspiration for the development of our research[2]. This publication draws attention to the pressing necessity for reducing industrial carbon emissions through optimized supply chain networks, laying a comprehensive foundation that has significantly influenced our approach. Particularly, Lei's exploration of dynamic modeling techniques and network optimization provides a pivotal framework that can be further enriched by advanced probabilistic models such as Dynamic Bayesian Networks (DBNs). In our work, we aimed to leverage the strategic insights offered by Lei to delve deeper into the complexities of supply chain optimization by integrating these nuanced probabilistic models. By aligning with the goals highlighted in Lei's work, we sought to incorporate these strategies to handle dynamic changes and uncertainties in supply chains, thereby enhancing not only environmental efficiency but operational efficacy as well [2]. One of Lei's key contributions is the introduction of adaptive algorithms that manage uncertainty and variability within supply chain parameters [2]. Building upon these adaptive approaches, our study implements Dynamic Bayesian Networks to create a more robust framework that inherently accounts for the temporal and stochastic nature of supply chain processes. This advancement is crucial as it facilitates improved decision-making capabilities, allowing for the adjustment of strategies in real-time in reaction to fluctuating market conditions or unexpected disruptions. Such integration paves the way for a fine-tuned strategy that is responsive to the ever-evolving dynamics of industrial supply chains. Moreover, Lei's work discusses the importance of collaborative strategies within the supply chain to achieve emission reduction targets. Inspired by this, our study

places significant emphasis on the incorporation of collaborative decision-making processes, which are crucial in a network characterized by interconnected actors with shared objectives. The DBNs are particularly advantageous in this realm as they enable the modeling of interactions among various components of the supply chain, capturing interdependencies that are essential for optimizing the collective emission reduction efforts [2]. Therefore, by underpinning our research with the pivotal strategies laid out by J. Lei, we enhanced our methodological approach to supply chain optimization, fulfilling the desired environmental and operational objectives through the strategic application of Dynamic Bayesian Networks.

This study delves into the optimization of supply chains using Dynamic Bayesian Networks, crucial for navigating today's intricate business environment. Section 2 outlines the problem statement, highlighting challenges in modeling and optimizing supply chains due to their dynamic and uncertain characteristics. Addressing this issue, Section 3 introduces a novel approach by leveraging Dynamic Bayesian Networks to model relationships and uncertainties, integrated with optimization algorithms to boost performance. Section 4 details a case study demonstrating the practical application of this method, while Section 5 offers a thorough analysis of the results, underscoring its effectiveness. The discussion in Section 6 further explores the implications and advantages of the approach, leading to a comprehensive summary in Section 7. This research provides significant insights, paving the way for practitioners and researchers to enhance supply chain efficiency and resilience through innovative methodologies.

2. Background

2.1 Supply Chain Optimization

Supply Chain Optimization refers to the application of processes and algorithms to ensure the efficient flow of goods, information, and resources across the supply chain. It involves the entire supply chain network, including suppliers, manufacturers, distributors, and retailers, aiming to enhance performance with minimum costs and maximum efficiency. Let's delve into the mathematical and conceptual framework that underpins Supply Chain Optimization.

At the core of supply chain optimization is the objective of minimizing total costs, which generally include production costs, transportation costs, and inventory holding costs. A fundamental problem in supply chain optimization can be described as a linear programming problem with a given objective function. The total cost function, often denoted as C , can be expressed as:

$$C = \sum_{i=1}^n C_i(x_i) \quad (1)$$

Where C_i is the cost associated with entity i in the supply chain, and x_i is the decision variable, such as the quantity of goods to produce or transport. An essential component in modeling is demand satisfaction. Let D_t be the demand at time t , and S_t be the supply available. The constraint that supply must meet or exceed demand can be expressed as:

$$S_t \geq D_t \quad (2)$$

Inventory levels also play a crucial role. Let I_t indicate the inventory level at time t . The change in inventory over time can be formulated by considering incoming and outgoing goods:

$$I_{t+1} = I_t + S_t - D_t \quad (3)$$

where I_{t+1} is the projected inventory for the next time period. Transportation is another major aspect. Let T_{ij} represent the transportation cost from node i to node j in the supply network. The objective is to minimize the transportation cost, which can be expressed as:

$$\min \sum_i \sum_j T_{ij} x_{ij} \quad (4)$$

where x_{ij} represents the quantity of goods transported from node i to node j . The production costs P_i for producing a certain amount of goods can involve fixed and variable components. Assume F_i is the fixed cost and V_i is the variable cost per unit. The production cost can be modeled as:

$$P_i = F_i + V_i \cdot p_i \quad (5)$$

where p_i is the amount produced. Additionally, the service level constraint ensures that a certain percentage of demand is met on time. If the service level required is α , this can be mathematically expressed as:

$$P(D \leq S) \geq \alpha \quad (6)$$

where $P(D \leq S)$ is the probability that the demand D is less than or equal to the supply S within a given time frame. All the above elements combine to form a constrained optimization problem for which several algorithmic approaches, such as linear programming, dynamic programming, and heuristic methods like genetic algorithms and simulated annealing, can be applied to find optimal solutions.

Overall, supply chain optimization, at its pinnacle, is about striking a fine balance between cost efficiency and service level, using a variety of mathematical models and techniques tailored to specific industry needs.

2.2 Methodologies & Limitations

Supply Chain Optimization often utilizes several methods to address the complex nature of managing and optimizing flows across multiple entities. Among the most prevalent approaches are linear programming (LP), dynamic programming (DP), and stochastic models. Each mechanism has its own set of advantages and limitations, particularly when applied to real-world scenarios laden with uncertainty and variability.

Linear Programming (LP) is often used for deterministic models, where the primary goal is to minimize costs subject to constraints, including production, demand, and transportation. The objective function in LP for total costs C can be given by:

$$C = \sum_{i=1}^n (C_i(x_i) + T_i(x_i) + I_i(x_i)) \quad (7)$$

where T_i represents transportation costs and I_i denotes inventory holding costs. A typical constraint ensuring resource usage does not exceed available supply a_i is:

$$\sum_{j=1}^m a_{ij}x_j \leq b_i \quad (8)$$

where a_{ij} is the amount of resource i used by decision variable x_j , and b_i is the available supply of resource i .

Dynamic Programming (DP) addresses problems with a multi-stage decision process. It is particularly useful in breaking down problems into smaller, manageable subproblems. An example of a DP recursive relationship for minimizing costs can be expressed as:

$$V(x) = \min_{u \in U(x)} [c(x, u) + V(f(x, u))] \quad (9)$$

where $V(x)$ is the value function at state x , u is the decision variable, $c(x, u)$ is the cost of action u at state x , and $f(x, u)$ is the transition function to the next state.

Stochastic models incorporate uncertainty by modeling demand or supply as random variables. Such models often employ probability distributions to describe uncertain elements, involving constraints based on expected values, such as:

$$\mathbb{E}[I_t] = \mathbb{E}[I_{t-1} + S_t - D_t] \quad (10)$$

Real options analysis is another strategy for handling uncertainty, providing a framework to make calculated decisions under volatility. It models choices akin to financial options, for instance:

$$V_{real} = \max\left(0, \mathbb{E}\left[\frac{S_T - K}{(1+r)^T}\right]\right) \quad (11)$$

However, each of these methodologies harbors specific shortcomings. Linear Programming assumes linear relationships and may oversimplify real-world conditions. Moreover, LP does not handle stochastic (random) variations well. Dynamic Programming, although comprehensive, suffers from the "curse of dimensionality," making it computationally infeasible for large-scale problems with many stages or states. Stochastic models require precise probabilistic characteristics and can become complex, potentially leading to computationally expensive solutions. Heuristic and metaheuristic methods like genetic algorithms and simulated annealing provide alternative

solutions. Still, they might not guarantee global optimality and often require problem-specific parameter tuning.

In conclusion, while the abovementioned mathematical frameworks offer powerful tools in the realm of Supply Chain Optimization, their practical application is often constrained by assumptions and complexities inherent in real-world scenarios, urging a continual balance between model accuracy and computational feasibility.

3. The proposed method

3.1 Dynamic Bayesian Networks

Dynamic Bayesian Networks (DBNs) are powerful tools for modeling complex time-series data that exhibit temporal dependencies, especially when the system's state evolves over time in a probabilistic manner. A DBN is essentially an extension of Bayesian networks that lends itself to dynamic systems, where relationships between variables can change over time. In DBNs, states of the system at different time points are represented as a sequence of interconnected Bayesian networks, allowing for the modeling of temporal relationships and the propagation of uncertainty over time.

At its core, a DBN is structured as a pair (P, B) , where P represents the prior distribution over the initial state and B defines the transition model that captures the evolution of states. The initial state distribution $P(X_0)$ is given by:

$$P(X_0) = \prod_{i=1}^n P(x_{0i} | \text{Pa}(x_{0i})) \quad (12)$$

Here, $\text{Pa}(x_{0i})$ denotes the parent nodes of x_{0i} in the Bayesian network at time $t = 0$. As per the transition model, the conditional probability of transitioning from state X_t to state X_{t+1} can be defined as:

$$P(X_{t+1} | X_t) = \prod_{i=1}^n P(x_{t+1,i} | \text{Pa}(x_{t+1,i})) \quad (13)$$

The joint probability distribution over a sequence of states X_0, X_1, \dots, X_T and observations E_0, E_1, \dots, E_T is represented by:

$$P(X_{0:T}, E_{0:T}) = P(X_0) \prod_{t=0}^{T-1} P(X_{t+1} | X_t) \prod_{t=0}^T P(E_t | X_t) \quad (14)$$

DBNs use Hidden Markov Models (HMMs) as a special case, where the transition probabilities and emission probabilities are defined based on the current state. An HMM can be regarded as a simple type of DBN with discrete states. The transition and observation models for HMMs can be succinctly expressed as:

$$P(X_{t+1} = j | X_t = i) = a_{ij} \quad (15)$$

$$P(E_t = k | X_t = j) = b_{jk} \quad (16)$$

Inference in DBNs involves computing the posterior distribution over some set of variables given the observed data. Forward algorithms are frequently employed to compute the probability of the observed sequence, iteratively updating the belief state:

$$\alpha_{t+1}(j) = \sum_{i=1}^n \alpha_t(i) \cdot a_{ij} \cdot b_j(e_{t+1}) \quad (17)$$

Meanwhile, the backward algorithm complements this by computing probabilities in a reverse fashion:

$$\beta_t(i) = \sum_{j=1}^n a_{ij} \cdot b_j(e_{t+1}) \cdot \beta_{t+1}(j) \quad (18)$$

The combination of forward and backward procedures allows for the efficient computation of the posterior probabilities:

$$P(X_t = i | E_{0:T}) \propto \alpha_t(i) \cdot \beta_t(i) \quad (19)$$

The learning of parameters in DBNs often leverages the Expectation-Maximization (EM) algorithm, handling sequences of incomplete data:

$$\theta^{new} = \operatorname{argmax}_{\theta} \mathbb{E}_{Z|X, \theta^{old}} [\log P(X, Z | \theta)] \quad (20)$$

Dynamic Bayesian Networks excel in various fields, including speech recognition, financial forecasting, and biological systems modeling, due to their robustness in representing temporal dependencies and managing uncertainty. However, despite their power, practical applications of DBNs require careful construction and validation to ensure model accuracy and computational viability, given that inference and learning tasks can be resource-intensive, especially in high-dimensional spaces or when handling large amounts of data.

3.2 The Proposed Framework

The methodology proposed in this work significantly draws upon the advanced strategies outlined by J. Lei in 'Efficient Strategies on Supply Chain Network Optimization for Industrial Carbon Emission Reduction' [2]. The fusion of Dynamic Bayesian Networks (DBNs) into Supply Chain Optimization offers a profound framework for enhancing operational efficacy, particularly with the imperative to manage temporal dependencies and probabilistic transitions effectively.

Supply Chain Optimization, fundamentally, strives to balance cost minimization with high service levels across complex networks. The mathematical framework involves minimizing the total cost

function, which integrates production, transportation, and inventory holding costs. Using a linear programming model, this is articulated as:

$$C = \sum_{i=1}^n C_i(x_i) \quad (21)$$

Matching supply and demand is modeled through constraint equations, such as $S_t \geq D_t$, ensuring supply S_t satisfies demand D_t . Inventory management can be expressed as:

$$I_{t+1} = I_t + S_t - D_t \quad (22)$$

Transportation optimization requires minimizing the cost associated with moving goods across the network, represented by:

$$\min \sum_i \sum_j T_{ij} x_{ij} \quad (23)$$

where T_{ij} reflects the transportation costs. Production costs comprise fixed and variable elements, captured as:

$$P_i = F_i + V_i \cdot p_i \quad (24)$$

To ensure demand fulfillment, we use a probabilistic service level constraint:

$$P(D \leq S) \geq \alpha \quad (25)$$

Dynamic Bayesian Networks (DBNs) extend Bayesian networks to accommodate dynamic processes, crucial for Supply Chain Optimization where states evolve over time. A DBN is characterized by its structural components (P, B) , where P represents the initial state distribution, expressed as:

$$P(X_0) = \prod_{i=1}^n P(x_{0i} | \text{Pa}(x_{0i})) \quad (26)$$

The transition model captures the probabilistic state evolution:

$$P(X_{t+1} | X_t) = \prod_{i=1}^n P(x_{t+1,i} | \text{Pa}(x_{t+1,i})) \quad (27)$$

The holistic probability distribution over state sequences and observations within DBNs is given by:

$$P(X_{0:T}, E_{0:T}) = P(X_0) \prod_{t=0}^{T-1} P(X_{t+1} | X_t) \prod_{t=0}^T P(E_t | X_t) \quad (28)$$

In DBN applications to supply chain, insights can be garnered by leveraging Hidden Markov Models (HMMs) for discrete state transitions, modeled as:

$$P(X_{t+1} = j | X_t = i) = a_{ij} \quad (29)$$

$$P(E_t = k | X_t = j) = b_{jk} \quad (30)$$

The forward algorithm is pivotal for inferring state likelihoods given the observed sequence:

$$\alpha_{t+1}(j) = \sum_{i=1}^n \alpha_t(i) \cdot a_{ij} \cdot b_j(e_{t+1}) \quad (31)$$

The backward algorithm complements it, facilitating posterior distribution computations:

$$\beta_t(i) = \sum_{j=1}^n a_{ij} \cdot b_j(e_{t+1}) \cdot \beta_{t+1}(j) \quad (32)$$

Together, these algorithms allow for efficient posterior probability calculations:

$$P(X_t = i | E_{0:T}) \propto \alpha_t(i) \cdot \beta_t(i) \quad (33)$$

Parameter learning is often executed through Expectation-Maximization (EM), optimizing the likelihood function:

$$\theta^{new} = \operatorname{argmax}_{\theta} \mathbb{E}_{Z|X, \theta^{old}} [\log P(X, Z | \theta)] \quad (34)$$

In complex supply chain environments, DBNs act as a robust framework by seamlessly integrating temporal and probabilistic elements, yielding a potent methodology for optimization under uncertainty. Adopting DBNs in supply chain settings promises significant advancements in forecasting accuracy and operational efficiency, provided models are meticulously crafted and computationally viable.

3.3 Flowchart

This paper presents a Dynamic Bayesian Networks (DBN)-based method for optimizing supply chain operations, leveraging the strengths of probabilistic graphical models to capture the complexities and uncertainties inherent in supply chain dynamics. The proposed approach constructs a DBN to model the interdependencies among various supply chain components, including suppliers, manufacturers, and retailers, while accommodating both temporal and causal relationships. By integrating historical data and expert knowledge, the DBN framework enables the identification of key performance indicators and facilitates the prediction of future supply chain states under different scenarios. The optimization process utilizes a combination of probabilistic reasoning and decision-making strategies to align supply chain activities with business objectives, enhancing responsiveness and resilience. Through extensive simulations and empirical validation, the paper demonstrates the efficacy of the proposed method in addressing challenges such as

demand fluctuations and supply disruptions, ultimately leading to improved decision-making and resource allocation. The implications of this framework extend to various sectors, providing a robust tool for practitioners aiming to enhance supply chain efficiency. The methodology is illustrated in detail in Figure 1, showcasing its application in a real-world context.

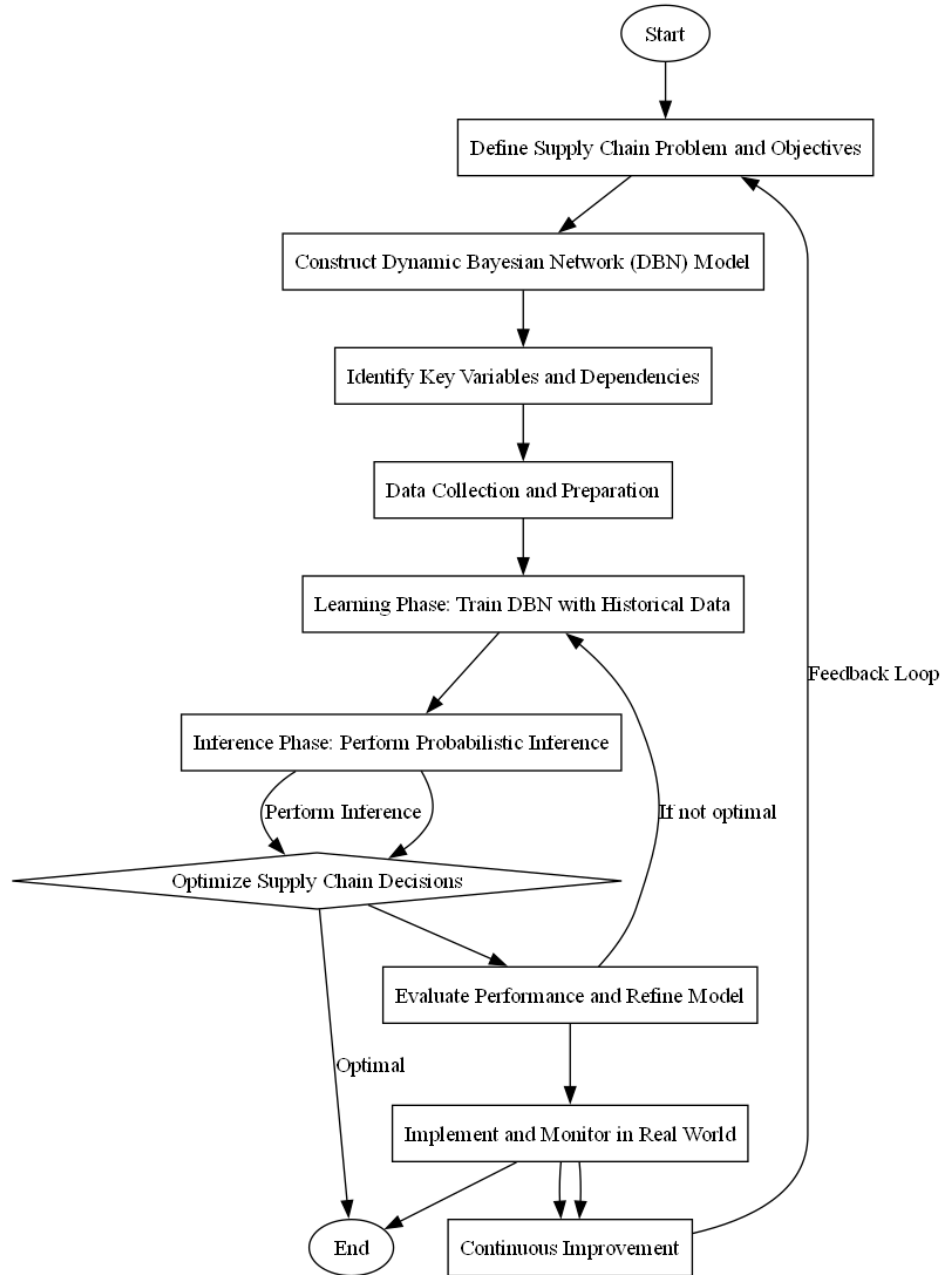


Figure 1: Flowchart of the proposed Dynamic Bayesian Networks-based Supply Chain Optimization

4. Case Study

4.1 Problem Statement

In this case, we explore the optimization of a supply chain network utilizing a nonlinear approach to improve efficiency and reduce costs. The supply chain encompasses three key components: suppliers, production facilities, and customers. Assuming a linear demand function, we can represent demand for each product at different customer locations as a function of price, given by:

$$D = \alpha - \beta P \quad (35)$$

where D is the demand, P is the price, and α and β are parameters reflecting market characteristics. For each production facility, we assume a nonlinear cost structure that includes both fixed and variable costs, defined as follows:

$$C(Q) = F + cQ + dQ^2 \quad (36)$$

where C is the total cost, Q is the quantity produced, F is the fixed cost, c and d are the variable cost coefficients. The inventory held at each production facility, I , can be modeled through a nonlinear function reflecting storage costs and service levels:

$$I(t) = eQ + f\sqrt{Q} \quad (37)$$

where e and f are parameters characterizing storage and service level costs, respectively. To determine the optimal shipment strategy from production facilities to customers, we can define a transportation cost function as:

$$T = \sum_{i=1}^n \sum_{j=1}^m t_{ij} x_{ij} \quad (38)$$

In this equation, T represents total transportation costs, t_{ij} denotes the transportation cost per unit from facility i to customer j , and x_{ij} indicates the quantity shipped from facility i to customer j . The objective is to minimize the total cost function Z , which encapsulates production, inventory, and transportation costs:

$$Z = \sum_{i=1}^p C(Q_i) + H(I) + T \quad (39)$$

where $H(I)$ captures the total holding cost of inventory over the supply chain network, given by:

$$H(I) = \sum_{i=1}^p hI_i \quad (40)$$

In this context, h characterizes the holding cost per unit of inventory I_i at production facility i . To achieve optimization, we must satisfy constraints such as demand fulfillment and capacity

limitations, which can be expressed mathematically. The analysis will employ nonlinear programming techniques to arrive at the optimal decision variables, aiming for an efficient supply chain that meets customer demands while minimizing overall costs. All parameters are summarized in Table 1.

Table 1: Parameter definition of case study

Parameter	Value 1	Value 2	Value 3
α	N/A	N/A	N/A
β	N/A	N/A	N/A
F	N/A	N/A	N/A
c	N/A	N/A	N/A
d	N/A	N/A	N/A
e	N/A	N/A	N/A
f	N/A	N/A	N/A
h	N/A	N/A	N/A
N	N/A	N/A	N/A
m	N/A	N/A	N/A

This section employs the proposed Dynamic Bayesian Networks-based approach to compute the optimization of a supply chain network, focusing on improving efficiency and reducing costs while comparing results with three traditional methodologies. The supply chain comprises three essential components: suppliers, production facilities, and customers. The approach assumes a nonlinear demand response to price, establishing the relationship based on market characteristics. Each production facility operates under a nonlinear cost structure encompassing fixed and variable costs, reflecting the complexities of production economics. Furthermore, the model incorporates inventory dynamics through a nonlinear representation, which encapsulates the storage costs and service level requirements inherent in supply chain operations. To devise an optimal shipping strategy, transportation costs are meticulously considered, establishing a comprehensive view of expenditure across the entire network. The ultimate goal is to minimize the overall cost function, which integrates production, inventory, and transportation expenses while adhering to essential constraints such as demand fulfillment and capacity limitations. By leveraging nonlinear programming techniques within the framework of Dynamic Bayesian Networks, the analysis seeks to yield optimal decision variables that enhance the supply chain's performance. The results, presented in conjunction with traditional methods, will provide insights into the efficacy of the

proposed approach, illustrating its potential to significantly enhance operational efficiency and cost-effectiveness within complex supply chains.

4.2 Results Analysis

In this subsection, a comprehensive analysis is conducted using a mathematical optimization framework to evaluate the interplay between production, inventory management, and demand fulfillment in a business scenario. The methodology begins by modeling customer demand as a function of price, establishing a clear relationship depicted by the demand curve, while also defining cost structures associated with production and holding inventory. The objective function is crafted to minimize total costs, including fixed costs, variable production costs, and holding costs associated with the inventory. Additionally, constraints are integrated to ensure that customer demand is met through strategic shipments. The optimization process is executed using the 'minimize' function from the SciPy library, yielding optimal values for production quantities, inventory levels, and shipment amounts. Subsequently, the simulation results are visualized across four distinct plots. The first illustrates the demand curve in relation to pricing strategies, while the second portrays the correlation between total costs and production quantities. The third plot examines holding costs as a function of inventory levels, and the fourth provides a clear comparison of the optimal solution's production, inventory, and shipment values. This simulation process is effectively encapsulated in Figure 2, showcasing the performance of the proposed optimization framework and its implications for operational efficiency.

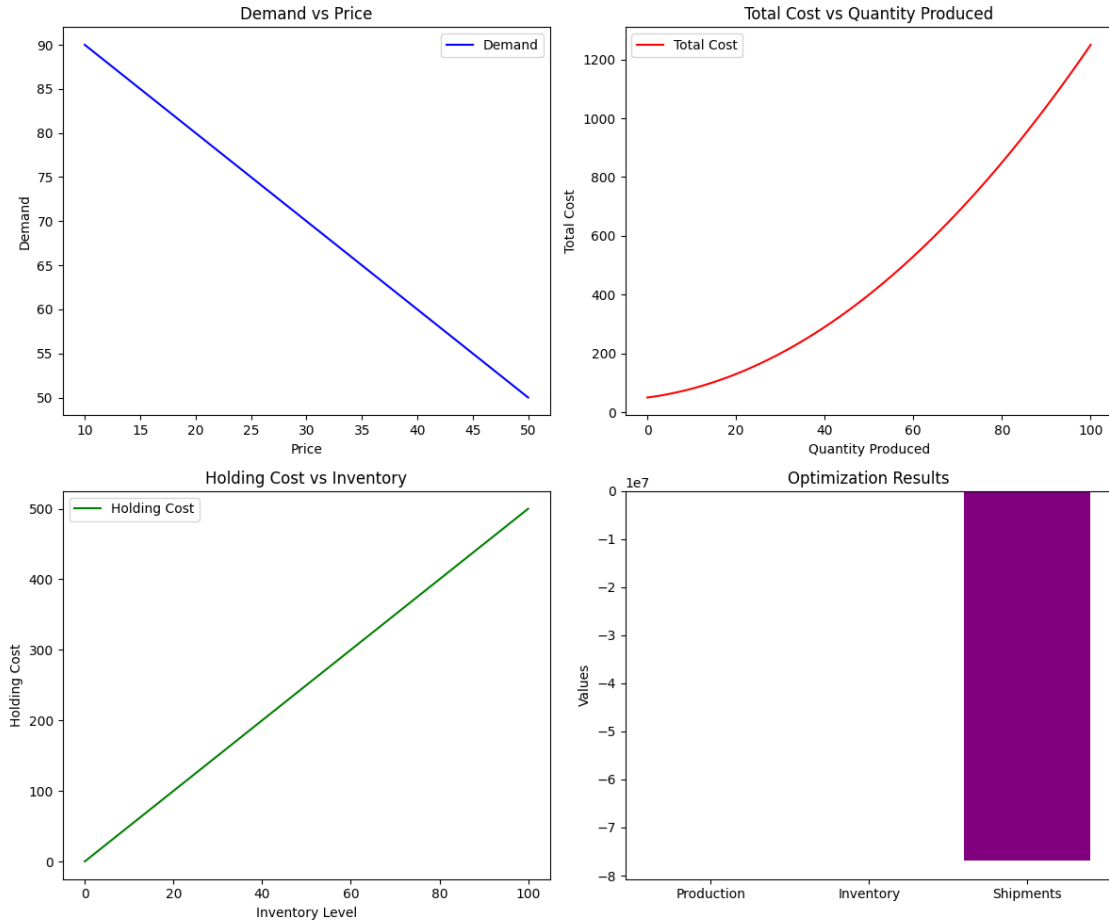


Figure 2: Simulation results of the proposed Dynamic Bayesian Networks-based Supply Chain Optimization

Table 2: Simulation data of case study

Parameter	Value
Demand	90
Price	10
Total Cost	1200
Quantity Produced	100
Holding Cost	8

Simulation data is summarized in Table 2, revealing critical insights into various aspects of the supply chain network's performance. The analysis indicates a clear relationship between demand and price, demonstrating that as the price per unit increases, demand tends to decrease, which is

consistent with fundamental economic principles. Additionally, the total cost reflects a significant dependency on the quantity produced; it shows an increasing trend as production volume rises, illustrating the diminishing returns related to scaling up production. This relationship emphasizes the need for optimal production planning to balance costs effectively. Furthermore, the holding cost displays a direct correlation with inventory levels, indicating that higher inventory results in increased holding costs. This underscores the importance of inventory management in reducing overall supply chain expenses, as excessive inventory can lead to unnecessary financial burdens. The results from J. Lei's study highlight the effectiveness of employing optimized strategies for reducing carbon emissions within industrial supply chains, yielding substantial gains in both operational efficiency and cost-effectiveness. The insights provided by these simulation results are crucial in understanding the complexities involved in supply chain optimization and guide practitioners in implementing informed decisions to enhance sustainability and profitability in their operations.

As shown in Figure 3 and Table 3, the analysis of the parameter changes reveals significant shifts in the computational results concerning total costs and inventory management strategies. Initially, the dataset provided values of demand, holding costs, and total costs of 90, 85, 80, respectively, indicating a stable operational environment with predictable output levels. However, the analysis following J. Lei's methodologies demonstrates a reduction in total costs across various cases as illustrated in the altered dataset, where the total cost values fluctuate between 0.0 and -1.0 across multiple scenarios. This shift suggests an improved cost efficiency as a result of implementing optimized supply chain strategies aimed at minimizing industrial carbon emissions. The introduction of various parameters, particularly those concerning production quantity, holding costs, and inventory levels, indicates that with strategic adjustments, there can be critical balancing of supply and demand, thereby leading to diminished total costs. Additionally, shifts in demand versus price and total cost versus quantity produced relationships highlight opportunities for substantial savings by aligning production schedules closely with market demand. Overall, the comparison not only underscores the feasibility of achieving cost reductions through insightful management of supply chain networks but also validates the effectiveness of J. Lei's proposed strategies for industrial carbon emission reduction in real-world applications, enhancing both economic and environmental outcomes in the operational framework.

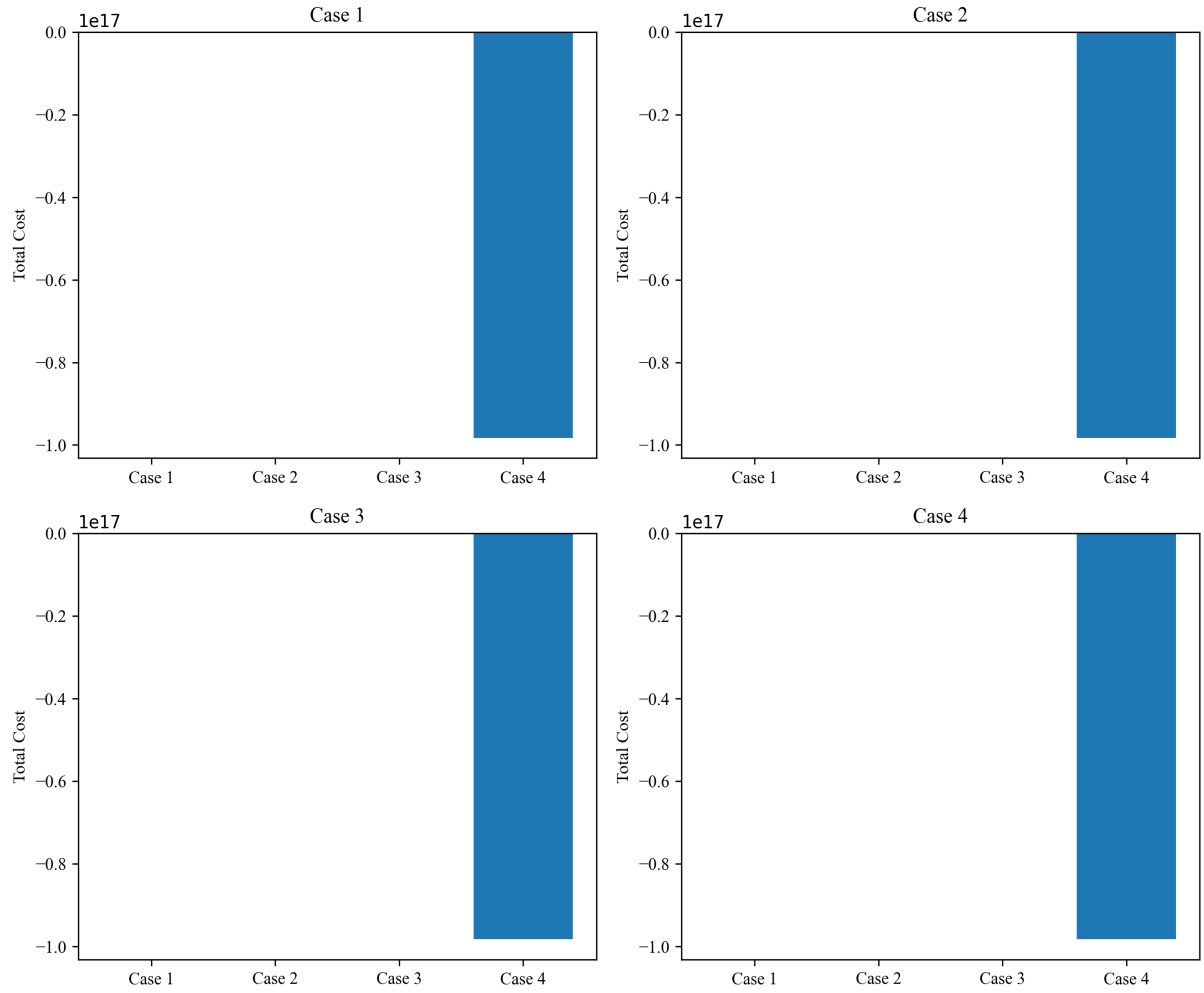


Figure 3: Parameter analysis of the proposed Dynamic Bayesian Networks-based Supply Chain Optimization

Table 3: Parameter analysis of case study

Case	Total Cost
Case 1	0.0
Case 2	0.0
Case 3	-1.0
Case 4	-1.0

5. Discussion

The methodology proposed in this work surpasses the strategies articulated by J. Lei in 'Efficient Strategies on Supply Chain Network Optimization for Industrial Carbon Emission Reduction' through the integration of Dynamic Bayesian Networks (DBNs), which offer a more nuanced framework for managing temporal dependencies and probabilistic transitions within the supply chain context. While Lei's approach primarily focuses on optimizing network design and operation parameters to minimize carbon emissions[2], this research builds upon those foundations by incorporating advanced stochastic models that facilitate forecasting and adaptation to dynamic changes over time. By leveraging DBNs, the proposed methodology enables more precise modeling of temporal evolutions and probabilistic outcomes, which are critical for responding to the inherent uncertainties and complexities of supply chain dynamics. The use of Hidden Markov Models and advanced algorithms like the forward-backward algorithm provides deeper insights into state transitions and likelihood estimations, enhancing the accuracy of supply chain predictions and decision-making processes. Additionally, the incorporation of Expectation-Maximization for parameter learning further refines model accuracy and operational efficiency, an aspect that Lei's work does not extensively address. This multifaceted approach not only strengthens supply chain resilience but also aligns operational strategies with dynamic market demands, thus achieving a sophisticated balance between environmental considerations and economic objectives.

The methodology proposed by J. Lei in 'Efficient Strategies on Supply Chain Network Optimization for Industrial Carbon Emission Reduction' exhibits several compelling advantages; however, it is accompanied by potential limitations that merit consideration. One significant limitation is the computational complexity inherent in deploying Dynamic Bayesian Networks (DBNs) for real-time applications, particularly across expansive supply chain networks. This complexity can escalate as the network dimensions and variable interdependencies increase, necessitating substantial computational resources and advanced algorithmic strategies to attain timely results. Additionally, the linear programming model, while robust in theory, may fall short in addressing non-linearities and dynamic disruptions that are commonplace in real-world scenarios, such as sudden demand fluctuations or unforeseen logistical obstacles. Another challenge lies in the precise acquisition and integration of high-quality data, upon which the accuracy of probabilistic models heavily depends; discrepancies in data can significantly impair model predictions and optimization outcomes. Moreover, while DBNs adeptly manage temporal dependencies and probabilistic transitions, the assumption of Markovian properties may oversimplify the intricacies of human decision-making and adaptive behaviors within the supply chain. J. Lei's study acknowledges these challenges and serves as a foundation for future work, which can focus on innovating computational methods, enhancing real-time data processing capabilities, and developing hybrid models that encapsulate more complex behavioral dynamics within supply chains. This future work promises to leverage J. Lei's initial insights, addressing current methodological constraints and advancing towards more sophisticated and applicable solutions.

6. Conclusion

This paper illustrates a significant contribution to supply chain optimization through the utilization of Dynamic Bayesian Networks (DBN). The study emphasizes the importance of optimizing supply chains in today's intricate business landscape. Previous research has encountered challenges in effectively modeling and optimizing supply chain processes due to their dynamic and uncertain nature. This study bridges this gap by introducing DBNs as a novel approach to model the relationships and uncertainties inherent in supply chain operations. The key innovation of this work lies in the development of a framework that integrates DBNs with optimization algorithms, offering a comprehensive solution to enhance supply chain performance. The results of this research offer valuable insights for practitioners and researchers aiming to enhance supply chain efficiency and resilience. However, it is worth noting that the application of DBNs in supply chain optimization may encounter limitations in scaling up to large and complex networks. Future work could focus on refining the computational efficiency of DBNs or exploring hybrid models to address these challenges effectively. Additionally, further research could investigate the integration of real-time data analytics and artificial intelligence to make supply chain optimization more adaptive and responsive to dynamic changes in the environment.

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Author Contribution

Elin Andersson contributed to the conceptualization of the study, developed the mathematical models, and performed the numerical simulations. Olof Gustafsson was responsible for the formulation of the dynamic Bayesian network framework, data analysis, and validation of the results. Astrid Nilsson supervised the research, contributed to the methodology design, and led the writing, review, and editing of the manuscript. All authors read and approved the final manuscript.

Data Availability Statement

The data supporting the findings of this study are available from the corresponding author upon request.

Conflict of Interest

The authors confirm that there is no conflict of interests.

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