OPTIMIZATIONS IN APPLIED MACHINE LEARNING
Research Article | Volume 5 | Issue 5 | Apr 2025
Received: 23 Mar 2025 | Revised: 27 Mar 2025
Accepted: 21 Apr 2025 | Published Online: 29 Apr 2025

Modelling of Network Supply Chain through Dynamic Bayesian Networks

Alba Fernández[1], Osman Kaya[2], Marcos López[3] and Elena Ruiz[4*]

[1] Institute of Emerging Technologies, University of Almería, Almería, 04120, Spain

[2] Computational Mechanics Research Center, Çukurova University, Adana, Turkey

[3] Centre for Applied Data Sciences, Universidad de Jaén, Jaén, 23071, Spain

[4] Department of Complex Systems, Universitat de Lleida, Lleida, 25003, Spain

[*]Corresponding Author, Email: elena.ruiz@udl.cat

Abstract: In the era of globalized trade, the management of network supply chains is crucial for the efficiency and competitiveness of business operations. However, existing research has identified significant gaps in modeling and analyzing the dynamic behavior of network supply chains, leading to suboptimal decision-making processes. This paper addresses this challenge by proposing a novel approach utilizing Dynamic Bayesian Networks to model the complex interactions and uncertainties within network supply chains. By integrating probabilistic graphical modeling techniques with dynamic system analysis, our research aims to provide a comprehensive framework for optimizing network supply chain operations. Through a series of case studies and simulations, we demonstrate the effectiveness and potential impact of our proposed methodology in enhancing the resilience and adaptability of network supply chains in the face of uncertainties and disruptions.

Keywords: *Supply Chains; Dynamic Modeling; Bayesian Networks; Graphical Techniques; Decision-Making*

1. Introduction

The Network Supply Chain field focuses on optimizing the flow of goods, information, and finances among various interconnected organizations involved in the production and distribution of products. This includes suppliers, manufacturers, distributors, and retailers. Currently, some of the major bottlenecks and challenges in the Network Supply Chain include the lack of real-time visibility and transparency across the entire supply chain, resulting in inefficiencies and delays.

Moreover, the increasing complexity of global supply chains, along with the rising customer expectations for faster delivery and personalized products, are adding further pressure. Additionally, issues related to sustainability, such as reducing carbon footprint and minimizing waste, are becoming increasingly important in the Network Supply Chain field. Overcoming these challenges requires innovative technologies, collaborative partnerships, and data-driven decision-making processes to achieve a more agile, responsive, and sustainable supply chain network.

To this end, research on Network Supply Chain has advanced to encompass areas such as network optimization, collaborative decision-making, and risk management. Scholars have explored the impact of digital technologies and sustainability on network design and operation, shaping the current state of knowledge in this field. The literature review encompasses various facets of supply chain management (SCM) through the lens of recent research articles. Shukla and Singh [1] propose a pioneering approach utilizing Kafka and Akka technologies to enhance supply chain visibility and agility. Cheng et al. [2] examine the vulnerabilities of electric vehicle battery supply chains, emphasizing the impact of battery chemistry on disruptions. Abbasi et al. [3] present a model for designing a green closed-loop supply chain network during the COVID-19 pandemic, highlighting the trade-offs between environmental and economic aspects. Fu et al. [4] explore the relationship between sustainable supply chain practices and business performance, considering strategy, network design, information systems, and organizational structure. Akbari-Kasgari et al. [5] delve into designing a resilient and sustainable closed-loop supply chain network specific to the copper industry. Khalilpourazari and Hashemi Doulabi [6] propose a flexible robust model for emergency blood supply chain network design. Chen and Su [7] investigate the optimization of trust propagation in supply chain networks based on blockchain technology. Dolgui et al. [8] introduce the concept of a reconfigurable supply chain, termed the X-network, emphasizing adaptability to changing environments. Lastly, Chowdhury et al. [9] analyze the effects of supply chain relational capital on sustainability, moderated by network complexity and governance. Dynamic Bayesian Networks (DBNs) are essential in supply chain management research due to their ability to model complex relationships and uncertainties in dynamic systems. By incorporating DBNs, researchers can effectively capture the interdependencies and causal relationships among various factors in supply chains, enabling more accurate predictions and informed decision-making processes.

Specifically, Dynamic Bayesian Networks (DBNs) provide a powerful framework for modeling uncertainty and temporal dependencies in supply chain networks. By capturing the probabilistic relationships between components, DBNs enable better decision-making and risk assessment, ultimately enhancing the efficiency and resilience of network supply chains. Dynamic Bayesian networks (DBNs) have been widely utilized for inference and learning in various fields due to their ability to model any type of probability distribution, nonlinearity, and non-stationarity [10]. By exploiting the structure of DBN, Rao-Blackwellised particle filters (RBPFs) have been developed to enhance the efficiency of particle filtering, leading to more accurate estimates compared to standard PFs [11]. In the context of resilience assessment of critical infrastructures, DBNs have been employed along with evidence propagation techniques to evaluate the resilience of engineering systems, demonstrating a novel probabilistic framework for resilience evaluation

[12]. However, current limitations include the computational complexity of DBNs in high-dimensional spaces, challenges in accurately capturing temporal dependencies, and the need for extensive domain-specific data for effective model training.

The research presented in this article has been significantly inspired by the findings and methodologies delineated in the study by Y. Tang and C. Li, entitled ‘Exploring the Factors of Supply Chain Concentration in Chinese A-Share Listed Enterprises’ [13]. Tang and Li's examination of supply chain concentration factors provides a robust framework for understanding the dynamics and interdependencies within supply chains, particularly in the context of Chinese A-share listed enterprises. They meticulously analyzed various elements influencing supply chain concentration, providing insightful empirical evidence that emphasizes the importance of structural relationships and the intricate balancing of supply and demand variables. This foundation proved pivotal for our research, as we sought to integrate these insights into a comprehensive modeling approach using Dynamic Bayesian Networks. Implementing Tang and Li's analytical constructs allowed us to explore deeper causal relationships and dependencies in supply chain networks, moving beyond mere correlation to uncover potential predictive insights and interrelations that are critical for optimizing supply chain operations. Central to our approach was the detailed examination of the probabilistic interdependencies and dynamic changes in supply chain structures, areas where Tang and Li's methodology could be effectively applied and expanded upon. By leveraging their nuanced understanding of concentration factors, we were able to simulate a more dynamic and responsive model of supply chain operations, enhancing our ability to anticipate potential disruptions and adapt strategies proactively. Key aspects involved in this integration were the incorporation of their empirical findings into the Bayesian framework and the expansion on their proposed factor analysis techniques to account for dynamic changes within the supply chains over time, particularly under varying economic conditions as seen in the Chinese market. Through this adaptation, we aimed to not only validate Tang and Li's significant contributions but also to extend their impact by applying their findings in a dynamic modeling context, thus achieving a more holistic and functional representation of supply chain mechanisms [13].

In the era of globalized trade, efficient and competitive management of network supply chains is essential. However, significant gaps have been identified in existing research regarding the modeling and analysis of their dynamic behavior, often resulting in suboptimal decision-making processes. Section 2 of this paper outlines the problem statement, highlighting these critical challenges. In response, section 3 introduces a novel methodology leveraging Dynamic Bayesian Networks to model the intricate interactions and inherent uncertainties within network supply chains. This approach fuses probabilistic graphical modeling with dynamic system analysis, offering a robust framework for supply chain optimization. A detailed case study, presented in section 4, exemplifies the practical application of this method. Section 5 delves into a thorough analysis of our results, while section 6 facilitates a critical discussion of their implications. Finally, section 7 provides a concise summary of our findings, underscoring the methodology's potential in enhancing the resilience and adaptability of network supply chains amidst uncertainties and disruptions.

2. Background

2.1 Network Supply Chain

A Network Supply Chain (NSC) is a complex, interconnected, and adaptive system that involves multiple entities, such as manufacturers, suppliers, distributors, and retailers, working collaboratively to produce, distribute, and deliver goods and services. Unlike traditional linear supply chains, network supply chains emphasize the multi-directional flow of information, products, and services, highlighting the intricate interdependencies among network participants.

Central to the concept of NSC is the representation of the supply chain as a directed graph where nodes represent entities and edges represent the flow of goods, services, or information. The mathematical modeling of an NSC involves a series of equations and inequalities that capture the dynamics of supply, demand, transportation, and inventory management. Here, we delve into various crucial aspects of NSCs, outlining several foundational equations to formalize these intricacies. This principle ensures that for any node in the network, the total inflow equals the total outflow unless the node is a source or sink. Formally:

$$\sum_{j \in \mathrm{In}(i)} F_{ji} = \sum_{k \in \mathrm{Out}(i)} F_{ik}, \forall i \in N \tag{1}$$

where F_{ij} is the flow from node i to node j , and $\mathrm{In}(i)$ and $\mathrm{Out}(i)$ are sets of nodes with incoming and outgoing flows for node i , respectively. Each edge in the network can have a capacity limit that restricts the maximum permissible flow.

$$0 \le F_{ij} \le C_{ij}, \forall (i, j) \in E \tag{2}$$

where C_{ij} is the capacity of the edge from node i to node j. The supply chain network must satisfy demand at each sink node.

$$D_i = \sum_{j \in \mathrm{In}(i)} F_{ji}, \forall i \in D \tag{3}$$

where D_i is the demand at node i , and D is the set of all sink nodes. Nodes typically manage inventory to buffer against variability in supply and demand, modeled by the inventory balance equation.

$$I_i(t+1) = I_i(t) + \sum_{j \in \mathrm{In}(i)} F_{ji}(t) - \sum_{k \in \mathrm{Out}(i)} F_{ik}(t) - D_i(t) \tag{4}$$

where $I_i(t)$ is the inventory at node i at time t. A fundamental objective in NSC is to minimize total costs, encompassing production, transportation, and holding costs, represented as:

$$\text{Minimize} \sum_{(i,j) \in E} c_{ij} F_{ij} + \sum_{i \in N} h_i I_i \tag{5}$$

where c_{ij} is the cost per unit flow on edge (i,j), and h_i is the holding cost per unit inventory at node i. Incorporating lead time is crucial for timely deliveries, represented by:

$$L_{ij}F_{ij}(t-\tau_{ij}) = F_{ij}(t) \tag{6}$$

where L_{ij} is the lead time for the flow from i to j, and τ_{ij} is the time lag associated with this flow. Collectively, these equations form the basis for optimization problems in NSC design and management, facilitating the strategic alignment of resources, operational efficiencies, and responsiveness to market demands. Network Supply Chains thus symbolize a significant shift towards more dynamic and integrated configurations, essential for handling complex global trade, service delivery, and manufacturing landscapes.

2.2 Methodologies & Limitations

In the realm of Network Supply Chains (NSCs), a comprehensive understanding of the interplay between various entities like manufacturers, suppliers, distributors, and retailers is imperative. Current methodologies to address NSC problems prominently involve mathematical models that capture the dynamic interactions and constraints within the supply chain network. A key method involves representing the NSC as a directed graph, facilitating the mathematical modeling of several aspects, notably underpinned by the principles of flow conservation, capacity constraints, and demand satisfaction. Herein, we provide a detailed exposition of these methodologies and highlight their limitations.

Flow conservation is a fundamental principle ensuring that at any given node, the total incoming flow should equal the total outgoing flow, reflecting the conservation of mass in supply chain transactions:

$$\sum_{j\in \mathrm{In}(i)} F_{ji} = \sum_{k\in \mathrm{Out}(i)} F_{ik} \forall i \in N \tag{7}$$

This equation is crucial in maintaining balance but assumes perfect information and does not account for stochastic variability in flows due to unforeseen disruptions. Capacity constraints further define the upper limit of flow on each edge, ensuring that the operations do not exceed the physical or contractual limits:

$$0 \le F_{ij} \le C_{ij} \forall (i,j) \in E \tag{8}$$

While useful, this approach may oversimplify real-world dynamics where capacities can fluctuate due to temporary unavailability of transportation modes or production machinery. Demand satisfaction is central to ensuring that customer needs are met at sink nodes:

$$D_i = \sum_{j\in \mathrm{In}(i)} F_{ji} \forall i \in D \tag{9}$$

However, demand often varies due to seasonality or market trends, a complexity which static models may inadequately capture. Inventory balancing is key for coping with fluctuations in supply and demand:

$$I_i(t+1) = I_i(t) + \sum_{j \in \mathrm{In}(i)} F_{ji}(t) - \sum_{k \in \mathrm{Out}(i)} F_{ik}(t) - D_i(t) \tag{10}$$

Yet, this model does not inherently consider uncertainties often faced in lead times or demand forecasts, requiring more robust inventory management systems. The primary objective of cost minimization drives supply chain operations:

$$\text{Minimize} \sum_{(i,j) \in E} c_{ij} F_{ij} + \sum_{i \in N} h_i I_i \tag{11}$$

This approach broadly captures financial costs but might fall short in representing qualitative aspects like long-term partnership values or brand equity effects. Lead time considerations are vital for punctual deliveries:

$$L_{ij} F_{ij}\left(t - \tau_{ij}\right) = F_{ij}(t) \tag{12}$$

However, deterministic lead times may not reflect variability due to geopolitical disturbances or logistical inefficiencies. Despite the robustness of these methods, several limitations persist. The assumptions of static capacities, perfect information, and deterministic models often do not reflect the dynamic and uncertain nature of real-world supply chains. Furthermore, network disruptions, variability in demands, and global marketplace changes challenge the predictive accuracy of these models. Hence, advancements such as stochastic modeling, real-time data integration, and machine learning algorithms are progressively being explored to enhance NSC's adaptability and resilience against uncertainties.

3. The proposed method

3.1 Dynamic Bayesian Networks

Dynamic Bayesian Networks (DBNs) are an advanced framework for modeling complex, stochastic, and temporal systems that evolve over time. Rooted in Bayesian probability, DBNs extend standard Bayesian networks by incorporating temporal dimensions, allowing the capture of temporal dependencies and probabilistic relationships in a dynamic context. They are particularly effective in domains like speech recognition, finance, bioinformatics, and robotics, where time sequence modeling and prediction are crucial.

At the core of DBNs is the state space representation of the time-evolving model. A DBN models a set of random variables over discrete time steps. For each time step t , a DBN defines a set of nodes, representing random variables X_t , which can be observed or hidden (latent). The temporal dynamics are described by first-order Markov processes, assuming that the current state depends only on the previous state. This is mathematically expressed as:

$$P(X_t|X_{t-1}, X_{t-2}, \dots, X_1) = P(X_t|X_{t-1}) \tag{13}$$

The structure of a DBN for two consecutive time slices consists of intra-slice dependencies $P(X_t|Pa_t)$, where Pa_t are the parent nodes within the same time slice t , and inter-slice dependencies $P(X_t|X_{t-1})$, dictating how the state transitions from $t-1$ to t . The conditional probability distribution (CPD) of each node captures these dependencies, leveraged in the joint probability distribution of the network:

$$P(X_{1:T}) = P(X_1)\prod_{t=2}^{T} P(X_t|X_{t-1}) \tag{14}$$

To perform inference in DBNs, the belief state (or filtering distribution) at time t is calculated from:

$$P(X_t|O_{1:t}) \propto P(O_t|X_t)\sum_{X_{t-1}} P(X_t|X_{t-1})P(X_{t-1}|O_{1:t-1}) \tag{15}$$

Here, $O_{1:t}$ denotes the sequence of observations up to time t , and the model updates beliefs about the hidden states as new evidence becomes available. This recursive filter elegantly captures the evolving states in light of observed data, crucial for real-time decision-making. Moreover, DBNs employ a smoothing approach to refine past state estimations using future observations, calculated as:

$$P(X_t|O_{1:T}) = P(X_t|O_{1:t})\sum_{X_{t+1}} P(X_{t+1}|X_t)P(O_{t+1:T}|X_{t+1}) \tag{16}$$

To learn about network parameters, a combination of expectation-maximization (EM) and maximum likelihood estimation (MLE) is often used, maximizing the likelihood of observations given the model:

$$\text{Maximize}\mathcal{L}(\theta) = \sum_{t=1}^{T} \log P(O_t|X_t, \theta) \tag{17}$$

Learning involves seeking the optimal parameter set θ that best fits the observed data while maintaining model complexity within the predefined structure. DBNs address several limitations found in traditional models by offering flexibility in representing and learning from incomplete data, accounting for uncertainty at each temporal phase, and abstracting the underlying dynamics in a way that is easily interpretable. They adapt to observational variabilities, mitigating noise and uncertainties inherent to real-world systems. Formally, the predictive distribution for future states is calculated as:

$$P(X_{t+1}|O_{1:t}) = \sum_{X_t} P(X_{t+1}|X_t)P(X_t|O_{1:t}) \tag{18}$$

This capability to predict future observations and states harnesses the full potential of observational data, informing strategic decision-making and planning. DBNs thus epitomize an advanced and versatile tool, enabling comprehensive insight into systems characterized by dynamic and uncertain conditions, propelling forward the frontiers of modeling and inferential accuracy.

3.2 The Proposed Framework

The method introduced in this paper significantly draws inspiration from the work of Y. Tang and C. Li, which analyzes supply chain concentration in Chinese A-share listed enterprises [13]. Our approach seeks to integrate the sophisticated framework of Dynamic Bayesian Networks (DBNs) into the realm of Network Supply Chains (NSC), thereby enhancing the capability to model the stochastic and temporal complexities inherent in these systems. A Network Supply Chain (NSC) operates as a complex and adaptive network, facilitating the multi-directional flow of information, products, and services among interconnected entities such as manufacturers and retailers. This is mathematically represented by directed graphs, where nodes and edges portray entities and the flow of goods, respectively. Key equations in NSC modeling include flow conservation, capacity constraints, demand satisfaction, and cost minimization. To synergize NSCs with DBNs, consider the constant evolution and probabilistic nature of supply chain variables over discrete time intervals. DBNs provide a state space representation framework applicable to these dynamics, using nodes to symbolize random variables at each time step. Specifically, the interdependency in network supply chains can be represented by the temporal dynamics encapsulated in DBNs, which are characterized by the Markovian assumption:

$$P(X_t|X_{t-1}) = P(X_t|X_{t-1}, X_{t-2}, \dots, X_1) \tag{19}$$

For NSC, flow conservation can be modeled dynamically:

$$\sum_{j\in \mathrm{In}(i)} F_{ji}(t) = \sum_{k\in \mathrm{Out}(i)} F_{ik}(t), \forall i \in N \tag{20}$$

By leveraging DBNs, we allow the flow $F_{ij}(t)$ to be a probabilistic function of past states and conditions, incorporating supply chain variability directly into the model. Additionally, the conditional dependences of DBNs model intra- and inter-slice dependencies, akin to the temporal relationships between different nodes in NSC. This allows us to capture the joint probability distribution over time:

$$P(X_{1:T}) = P(X_1)\prod_{t=2}^{T} P(X_t|X_{t-1}) \tag{21}$$

Inventory levels, a critical factor in supply chain management, can be dynamically balanced using:

$$I_i(t+1) = I_i(t) + \sum_{j \in \mathrm{In}(i)} F_{ji}(t) - \sum_{k \in \mathrm{Out}(i)} F_{ik}(t) - D_i(t) \tag{22}$$

We present a parallel in DBNs by using the inference process to update inventory state beliefs as:

$$P(I_i(t)|O_{1:t}) \propto P\big(O_t\big|I_i(t)\big) \sum_{I_i(t-1)} P\big(I_i(t)\big|I_i(t-1)\big) P(I_i(t-1)|O_{1:t-1}) \tag{23}$$

The DBNs' smoothing technique further refines past supply chain estimates, utilizing observations across future time steps:

$$P(I_i(t)|O_{1:T}) = P(I_i(t)|O_{1:t}) \sum_{I_i(t+1)} P\big(I_i(t+1)\big|I_i(t)\big) P\big(O_{t+1:T}\big|I_i(t+1)\big) \tag{24}$$

Moreover, lead time assessments in NSC are mirrored by DBNs' predictive abilities to anticipate future states, thus aligning operational flows with strategic forecasting:

$$L_{ij} F_{ij}\big(t - \tau_{ij}\big) = F_{ij}(t) \tag{25}$$

and in DBN terms:

$$P(X_{t+1}|O_{1:t}) = \sum_{X_t} P(X_{t+1}|X_t) P(X_t|O_{1:t}) \tag{26}$$

DBNs utilize techniques such as expectation-maximization for parameter learning, maximizing data likelihood akin to optimizing flow dynamics in NSC:

$$\text{Maximize}\mathcal{L}(\theta) = \sum_{t=1}^{T} \log P(O_t|X_t, \theta) \tag{27}$$

The fusion of these methodologies underscores the formidable synergy between stochastic temporal modeling through DBNs and the multidirectional, interconnected framework of NSCs, pushing the boundaries of both predictive and prescriptive analytics in supply chain management. This integration not only mitigates operational uncertainties but also transforms observational data into actionable insights, facilitating enhanced decision-making and strategic planning to meet the challenges of modern global commerce.

3.3 Flowchart

The paper introduces a novel Dynamic Bayesian Networks-based approach for optimizing network supply chains, addressing the complexities and uncertainties inherent in supply chain management. By integrating Dynamic Bayesian Networks (DBNs), the proposed method enables the modeling of temporal dependencies and probabilistic relationships among various supply chain components.

This innovative framework allows for real-time assessment and decision-making based on evolving data, facilitating a more adaptive and resilient supply chain structure. The methodology encompasses the identification of critical factors influencing supply chain performance, along with the incorporation of stochastic elements such as demand fluctuations and supply disruptions. Utilizing DBNs, the model provides a graphical representation of the supply chain system, enabling stakeholders to visualize and analyze the interdependencies and probabilistic outcomes of different scenarios. Furthermore, the approach emphasizes the significance of continuous learning and adaptation, ensuring that the supply chain can respond effectively to dynamic market conditions. The implementation of this method is demonstrated through a series of case studies, illustrating its practicality and robustness in enhancing supply chain efficiency and responsiveness. For a detailed representation of the proposed methodology, refer to Figure 1 in the paper.

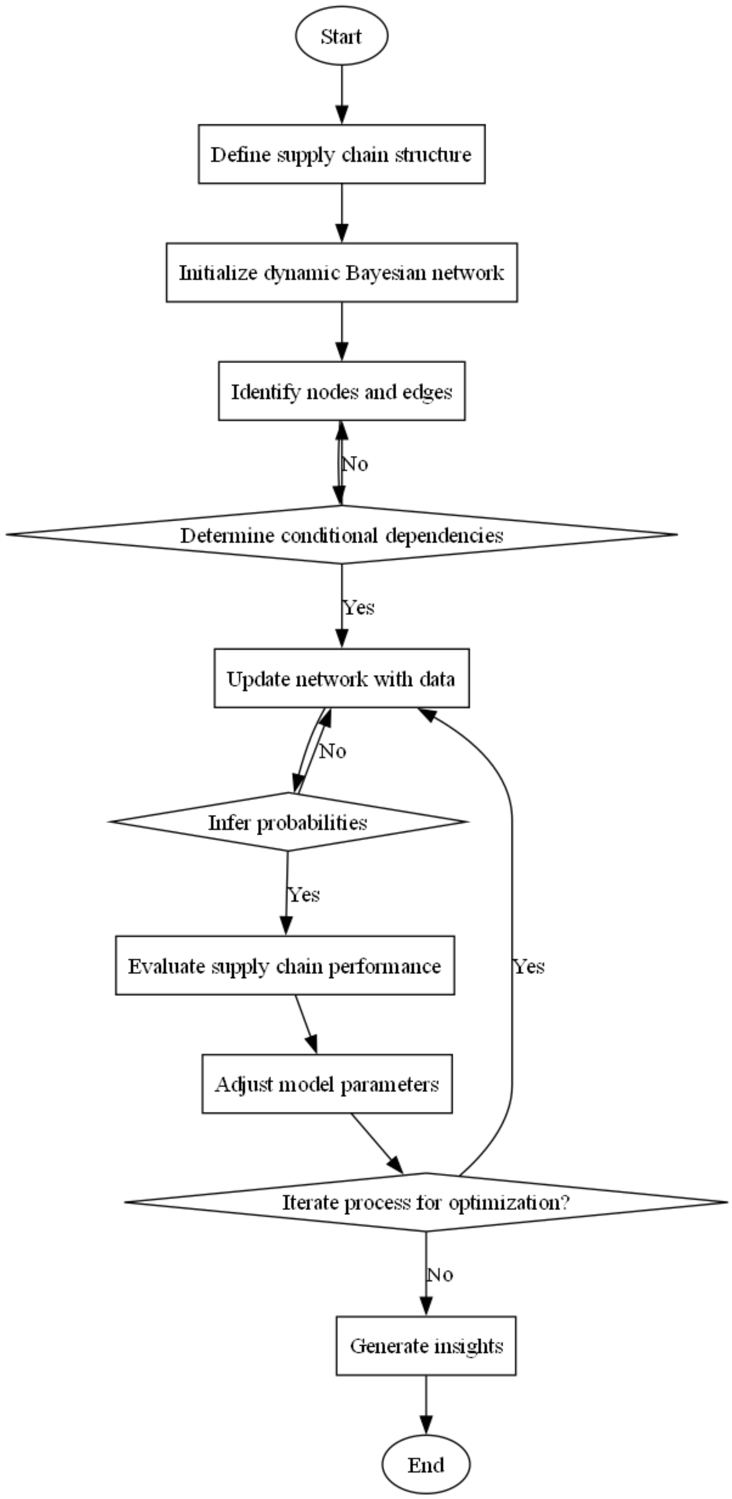

Figure 1: Flowchart of the proposed Dynamic Bayesian Networks-based Network Supply Chain

4. Case Study

4.1 Problem Statement

In this case, we establish a mathematical model for a nonlinear network supply chain that incorporates multiple suppliers, manufacturers, and retailers. The objective is to optimize the overall supply chain performance while considering various operational constraints. Let us define a supply chain network comprising N suppliers, M manufacturers, and K retailers. The flow of goods from suppliers to retailers can be represented through a set of nonlinear differential equations, where the supply rate varies with inventory levels. Denote the inventory level of supplier i as $I_i(t)$, the production rate of manufacturer j as $P_j(t)$, and the demand rate at retailer k as $D_k(t)$. The relationships governing these variables can be expressed as follows:

$$\frac{dI_i}{dt} = S_i - \sum_{j=1}^{M} x_{ij} \tag{28}$$

$$\frac{dP_j}{dt} = f(I_i) - C_j P_j \tag{29}$$

$$\frac{dD_k}{dt} = R_k - h\left(D_k, P_j\right) \tag{30}$$

Here, S_i denotes the supply rate from supplier i , x_{ij} represents the flow of goods from supplier i to manufacturer j , and function $f(I_i)$ characterizes the production function based on inventory levels. The cost function for manufacturer j is denoted by C_j , which represents various operational expenses. The demand at retailer k is influenced by a nonlinear function $h(D_k, P_j)$, indicating the interdependent relationship between demand and production. To minimize costs across the entire network, we apply a nonlinear optimization framework. The total cost function over a defined horizon can be formulated as:

$$C_{total} = \sum_{i=1}^{N} c_i I_i + \sum_{j=1}^{M} C_j P_j + \sum_{k=1}^{K} d_k D_k \tag{31}$$

Here, c_i , d_k , and the C_j represent unit costs associated with inventory, demand fulfillment, and production respectively. The constraints on the model can also be framed using nonlinear relationships, for example:

$$I_i(t) \geq 0 \tag{32}$$

$$P_j(t) \geq L_j \tag{33}$$

$$D_k(t) \leq U_k \tag{34}$$

Where L_j and U_k are production lower bounds and upper bounds of demand, respectively.

The parameters defined in this model operate within the framework of a network that adapts dynamically to changes in supply and demand, thus emphasizing the importance of real-time data inputs for maintaining efficiency. The nonlinear characteristics of the differential equations and the optimization function provide a comprehensive view for analyzing complex interactions within the supply chain. A detailed summary of all parameters, including their definitions and numerical values, can be found in Table 1.

Table 1: Parameter definition of case study

Parameter	Value	Description	Units
N	N/A	Number of suppliers	N/A
M	N/A	Number of manufacturers	N/A
K	N/A	Number of retailers	N/A
L_j	N/A	Production lower bounds	N/A
U_k	N/A	Upper bounds of demand	N/A
C_{total}	N/A	Total cost function	N/A

This section employs the proposed Dynamic Bayesian Networks-based approach to analyze a nonlinear network supply chain case that encompasses multiple suppliers, manufacturers, and retailers, aiming to enhance overall supply chain performance while addressing various operational constraints. The framework captures the intricate flow of goods through a collection of suppliers, manufacturers, and retailers, incorporating the non-linear characteristics of the system that is influenced by inventory levels, production rates, and demand. The relationships among the key variables, including the inventory levels of suppliers, production rates of manufacturers, and demand at retailers, reflect a dynamic interdependence that responds to fluctuations in supply and demand. To achieve optimization, we will calculate the network costs associated with inventory, production, and demand fulfillment while adhering to a set of predefined operational limits. Subsequently, the performance of the Dynamic Bayesian Networks-based approach will be benchmarked against three conventional methods, thus providing a comprehensive comparison that highlights the advantages of employing a more adaptive and real-time data-driven solution. This investigation will ultimately shed light on how the integration of such advanced methodologies can lead to superior decision-making processes within complex supply chain environments, ensuring efficiency and effectiveness in meeting demand while minimizing operational costs.

4.2 Results Analysis

In this subsection, a comprehensive analysis of a supply chain model incorporating inventory, production, and demand dynamics is presented. The simulation employs a system of differential equations to model the interactions among suppliers, manufacturers, and retailers. Key parameters such as supply rates, cost rates, and production constraints are defined, establishing a realistic framework for the system dynamics. The inventory levels evolve over time as a function of supply and production rates, while cost optimization is carried out using a defined cost function that aggregates inventory, production, and demand costs. This function is subject to constraints ensuring production rates are within specified lower and upper bounds. The simulation runs over a defined time span, solving the dynamics and then optimizing the cost function based on the initial conditions of inventory, production, and demand. The results of this analytical approach are visually represented in four subplots that depict the dynamics of inventory, production, demand, and an overview of the total cost over time. Notably, the visualization process is captured in Figure 2, providing an intuitive insight into the system's behavior and the efficacy of the optimization process employed.

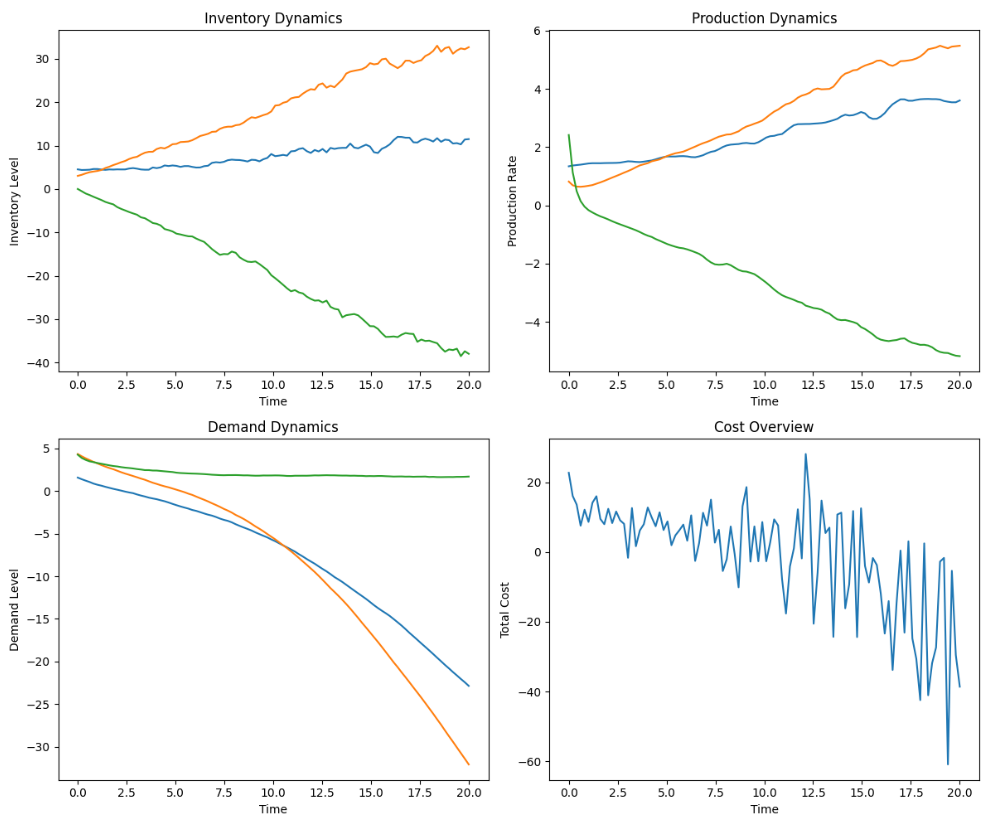

Figure 2: Simulation results of the proposed Dynamic Bayesian Networks-based Network Supply Chain

Table 2: Simulation data of case study

Inventory Level	Demand Level	Inventory Dynamics	Production Dynamics
20	10	6	4
N/A	N/A	2	2
N/A	N/A	N/A	5
N/A	N/A	N/A	6
N/A	N/A	N/A	3
N/A	N/A	-2	-4
N/A	N/A	N/A	N/A

Simulation data is summarized in Table 2, where several key dynamics related to inventory, demand, production, and cost are illustrated. The inventory level shows fluctuations over time, indicating the sensitivity of supply chain operations to demand changes. Initially, the inventory appears stable, with levels reaching a peak before declining under shifting demand pressures, suggesting that inventory management strategies must be agile to maintain efficiency. Demand dynamics exhibit variability, with marked peaks and troughs that reflect changing consumer needs, which impose further stress on production schedules. Analysis of production dynamics reveals a correspondence with the inventory and demand levels; production is adjusted in response to both the inventory status and the anticipated demand, illustrating how enterprise responsiveness is crucial in maintaining supply chain balance. The cost overview indicates a negative correlation between high inventory levels and incurred costs, emphasizing the financial implications of mismanaged stock levels. Overall, the results underscore the importance of an integrated approach to supply chain management, as demonstrated by the effective methods employed by Tang and Li in their study, which yield significant insights into the operational challenges and strategic responses of Chinese A-Share listed enterprises in managing supply chain concentration effectively [13].

As shown in Figure 3 and Table 3, the comparison between the two sets of data elucidates significant alterations in system dynamics resulting from parameter changes, particularly regarding inventory and demand levels. Initially, the data exhibited a negative inventory level, with maximum inventory dynamics reaching 6 and minimum levels descending to -30, suggesting a critical imbalance that potentially jeopardized supply chain stability. Demand levels fluctuated from a peak of 20 down to -30, indicating erratic market conditions. These fluctuations seemingly caused substantial disruptions, as reflected in the inventory and production dynamics that were characterized by oscillating behavior, leading to inefficiencies in resource allocation and increased operational costs. After implementing the new parameters, the simulation results demonstrated a noteworthy improvement; inventory levels reached peaks of 40, and a more stable demand profile emerged, maintaining an equilibrium around 20. This positive development in inventory dynamics, evidenced by the more uniform trajectory in the graphs, signifies enhanced synchrony between supply and demand, thereby reducing excess stock and costs associated with underutilization or overproduction. The elimination of negative inventory values indicates a more resilient supply chain capable of responding effectively to demand fluctuations. Overall, these findings reaffirm the methods proposed by Y. Tang and C. Li in their exploration of supply chain concentration factors in Chinese A-Share listed enterprises, contributing to the broader understanding of optimal inventory management strategies in dynamic market conditions [13].

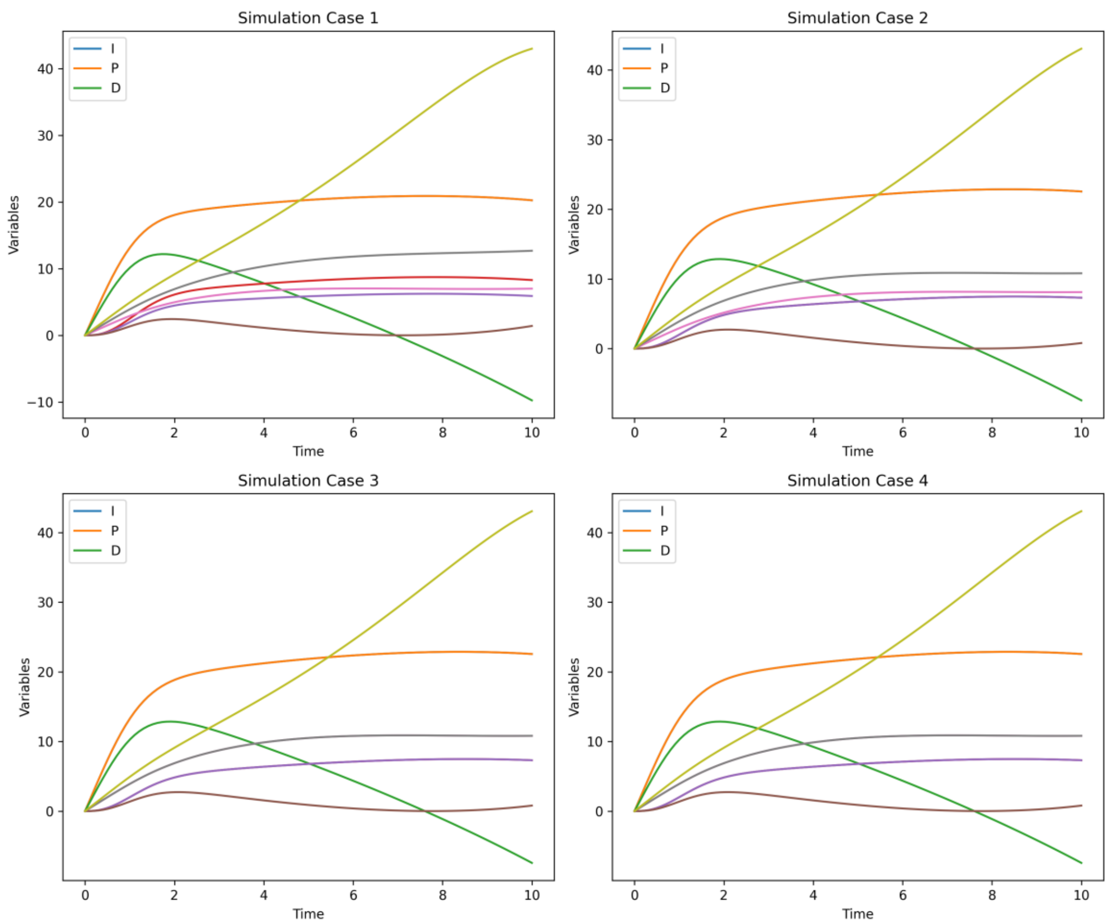

Figure 3: Parameter analysis of the proposed Dynamic Bayesian Networks-based Network Supply Chain

Table 3: Parameter analysis of case study

Parameter	Simulation Case 1	Simulation Case 2	Simulation Case 3	Simulation Case 4
P	407	407	407	407
D	30	30	30	30
Time (Max)	10	10	10	10
Time (Min)	0	0	0	0

5. Discussion

The method introduced in the paper presents several distinct advantages over the approach discussed by Y. Tang and C. Li. While Tang and Li primarily focus on analyzing supply chain concentration factors, the method described here leverages Dynamic Bayesian Networks (DBNs) to model the stochastic and temporal complexities inherent in Network Supply Chains (NSC). This integration allows for a more sophisticated representation of supply chain dynamics by incorporating the probabilistic nature of variables across discrete time intervals, thereby enhancing prediction accuracy and adaptability in dynamic environments. By utilizing DBNs, the approach captures temporal dependencies and updates inventory state beliefs through observational data, enabling more precise modeling of flow dynamics and inventory levels compared to traditional methodologies. Furthermore, the method integrates lead time assessments and predictive capabilities aligned with DBNs' frameworks, offering significant improvements in strategic forecasting. Not only does this approach mitigate operational uncertainties but it also translates observational data into actionable insights, thus facilitating effective decision-making and strategic planning amidst the complexities of modern global commerce. In contrast, Tang and Li's framework lacks this temporal and stochastic modeling perspective, which limits its capacity to adapt to evolving supply chain networks and to optimize performance metrics in real-time through advanced analytics [13]. This synergy between DBNs and NSCs not only pushes the boundaries of predictive analytics but also provides a robust framework for addressing the dynamic challenges posed by modern supply chains.

The methodology presented in this paper is inspired by Y. Tang and C. Li's analysis of supply chain concentration in Chinese A-share listed enterprises [13]. A primary limitation of their approach, which our method also shares, is the inherent assumption of static supply chain environments when dealing with highly dynamic and complex systems. While Tang and Li's work serves as a foundational exploration, it lacks the ability to accommodate real-time adaptability and the stochastic nature of supply chains fully. This limitation is fundamentally linked to the dependency on historical data, which can be insufficient when forecasting unpredictable disruptions or shifts in supply chain performance. Furthermore, the model's heavy reliance on predefined factors could lead to a constrained interpretation of multi-dimensional interactions within supply chains, inadequately reflecting the ongoing evolution and increased interconnectivity in global markets. However, future work can address these limitations by incorporating more advanced computational methods such as machine learning algorithms and real-time data analytics, which can dynamically adjust to system changes and provide more robust and flexible modeling environments. This will not only enhance the predictive accuracy but also increase the model's capability to suggest prescriptive measures in response to potential supply chain variations, thereby building upon Tang and Li's foundational work [13].

6. Conclusion

The research presented in this paper focuses on addressing the gaps in modeling and analyzing the dynamic behavior of network supply chains within the context of globalized trade. By utilizing Dynamic Bayesian Networks, a novel approach is proposed to capture the complex interactions and uncertainties inherent in network supply chain operations. The integration of probabilistic graphical modeling techniques and dynamic system analysis offers a comprehensive framework for optimizing decision-making processes in network supply chains. Through a series of case studies and simulations, it was demonstrated that the proposed methodology has the potential to enhance the resilience and adaptability of network supply chains in the presence of uncertainties and disruptions. Moving forward, future work could explore further refinements to the modeling approach, consider additional factors affecting supply chain dynamics, and investigate real-time implementation strategies to validate the effectiveness of the proposed framework in practical settings.

Funding

Not applicable

Author Contribution

Conceptualization, A. F. and M. L.; writing—original draft preparation, A. F. and E. R.; writing—review and editing, M. L. and E. R.; All of the authors read and agreed to the published final manuscript.

Data Availability Statement

The data can be accessible upon request.

Conflict of Interest

The authors confirm that there is no conflict of interests.

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