



# An Efficient Probabilistic Decision Tree-guided Approach for Battery Life Estimate

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**Abstract:** Battery life estimation is crucial for the optimal operation of various electronic devices and renewable energy systems. However, existing methods often suffer from limitations in accuracy and computational efficiency. This paper addresses the current challenges by proposing an innovative Probabilistic Decision Tree-guided approach for battery life estimation. The proposed method leverages the power of decision trees to efficiently model the complex relationships between battery usage patterns and degradation factors, while incorporating probabilistic techniques for uncertainty quantification. Through extensive experiments and comparisons with state-of-the-art methods, our approach demonstrates superior accuracy and computational efficiency, making it a promising solution for reliable battery life estimation in practical applications.

**Keywords:** *Battery Life Estimation; Probabilistic Decision Trees; Computational Efficiency; Uncertainty Quantification; Degradation Factors*

## 1. Introduction

Battery Life Estimate is a field of research focused on predicting the remaining operational time of batteries across various devices. The main challenge faced in this field is the complexity of battery behavior, influenced by factors such as usage patterns, temperature fluctuations, and aging effects. Accurately estimating battery life requires advanced modeling techniques and data analysis, often

hindered by the lack of standardized testing protocols and real-world validation. Additionally, the increasing demand for longer-lasting batteries in electronic devices further amplifies the need for more precise and reliable estimation methods. Enhancements in battery technology, coupled with interdisciplinary research efforts, are crucial in overcoming these hurdles and advancing the field of Battery Life Estimate.

To this end, current research on Battery Life Estimate has advanced to the stage where sophisticated machine learning algorithms are being utilized to accurately predict and optimize battery performance in various devices. Additionally, real-time monitoring technologies are also being integrated to provide timely insights into battery health and usage patterns. The literature review explores various methodologies for estimating the state of health (SOH) and remaining useful life of lithium-ion batteries, crucial for ensuring optimal battery performance and longevity in electric vehicles [1][2][3][4]. Arora et al. (2024) developed a time-temperature analysis algorithm to estimate lithium-ion battery useful life based on vehicle level testing, considering thermal degradation models and high ambient temperatures [1]. Yang et al. (2023) conducted a comprehensive review of SOH estimation strategies, highlighting experimental, model-based, and machine learning approaches, emphasizing the potential of a knowledge graph-based framework for battery data management [2]. Sangiri et al. (2022) proposed a novel methodology using discrete Fourier transformation to estimate the state-of-health and remaining-useful-life of lithium-ion batteries [3]. Additionally, prediction models for remaining useful life using electrochemical models, improved cycle aging cost models, and long short-term memory approaches were discussed [4][5][6]. Overall, the diverse studies contribute to enhancing battery performance, longevity, and management strategies [4]. The study explores methodologies for estimating the state of health and remaining useful life of lithium-ion batteries in electric vehicles. Using Probabilistic Decision Tree is crucial for its ability to provide probabilistic predictions that incorporate uncertainty, making it a valuable tool for optimizing battery performance and longevity. It offers a structured approach to decision-making, integrating multiple sources of information to enhance the accuracy of SOH and RUL estimations, thereby improving battery management strategies.

Specifically, Probabilistic Decision Trees (PDTs) enhance battery life estimation by incorporating uncertainty and variability in real-world conditions. By modeling the likelihood of different operational scenarios, PDTs provide more accurate predictions of battery performance, enabling better energy management and optimization in various applications. Literature review on probabilistic decision tree research: Probabilistic decision trees have been widely applied in various fields, such as wind power forecasting [7], multi-valued preference environment classification [8], lymphoid neoplasm diagnosis prediction [9], temporal data classification [10], and character recognition [11]. In the study by Khan et al., a new hybrid approach incorporating clustering and probabilistic decision trees was proposed for wind power forecasting on large scales [7]. Zhou et al. introduced machine learning methods utilizing probabilistic decision trees for classification under multi-valued preference environments [8]. Chong et al. developed a machine-learning expert-supporting system using a probabilistic decision tree algorithm for diagnosing lymphoid neoplasms [9]. Akhlagh et al. focused on temporal data classification and rule extraction employing a

probabilistic decision tree model [10]. Aulia explored the application of a probabilistic fuzzy decision tree in diagnosing coronary heart disease, achieving a high accuracy of 95% [12]. In addition, decision trees have also been combined with deep learning for character recognition applications [11]. Hawarah et al. addressed the issue of missing values in probabilistic decision trees during classification, contributing to improved data handling [13]. Mendonça et al. proposed a decision tree-based machine learning model for assessing the Basic Education Development Index, showcasing the significant impact of technology-related variables on educational quality [14]. Furthermore, Nandanwar et al. utilized a probabilistic fuzzy decision tree method for load management to enhance voltage security [15]. Overall, these studies demonstrate the versatility and effectiveness of probabilistic decision trees in addressing various challenges across different domains. However, limitations persist in the scalability of probabilistic decision trees to handle large datasets efficiently, their sensitivity to noise and outliers, and the potential for overfitting in complex classification tasks.

The insights derived from the work by W. Huang, Y. Cai, and G. Zhang have been invaluable in shaping the methodology we employed in our research. Their exploration into the utilization of sparse ridge regression provided a novel framework that allowed for more accurate modeling of battery degradation dynamics [16]. By implementing the sparse ridge regression approach, Huang et al. demonstrated how the identification of pertinent features influencing degradation could lead to enhanced predictive capabilities while simultaneously reducing the complexity of the model, which often results from handling large datasets with multifaceted variables. This methodology inspired us to delve deeper into probabilistic modeling techniques, considering how regression analyses could complement decision-making processes, particularly in systems characterized by uncertainty and variability intrinsic to battery performance metrics [16]. The approach taken by Huang and colleagues particularly underscored the importance of balancing interpretability and predictive accuracy, a nuance often compromised in overly complex models. They achieved this through the meticulous tuning of penalty parameters in sparse ridge regression, which we adapted in our work to fine-tune decision tree structures in probabilistic frameworks. This adaptation was not merely a transposition of methods but rather a synergistic integration that sought to leverage the strengths of regression analysis as a tool for feature selection, thereby guiding the construction of decision trees that are both concise and informative. Further, their treatment of degradation analysis as a multi-faceted problem encouraged us to think broadly about the array of factors that could impact system performance over time. By systematically narrowing down variables to those of utmost significance through sparse modeling, our research benefits from reduced computational overhead and increased focus on critical decision paths within the probabilistic modeling process. In essence, drawing upon the technical rigor and adaptive frameworks proposed by Huang, Cai, and Zhang, we have been able to create a model architecture that is not only reflective of the intricate dynamics governing battery longevity but also efficient in its predictive mandate. As we continue to refine and iterate on our methodologies, the foundational principles established by their work remain a testament to the enduring value of integrating advanced regression techniques within broader scientific inquiries aimed at optimizing energy systems [16].

In addressing the critical need for accurate battery life estimation, particularly for electronic devices and renewable energy systems, this paper highlights existing limitations in prevailing methods concerning accuracy and computational efficiency. Section 2 delineates the problem statement, pinpointing the challenges that have hindered advancements in this field. In Section 3, the paper introduces an innovative approach, employing a Probabilistic Decision Tree-guided methodology that adeptly models the intricate interplay between battery usage patterns and degradation factors, enhanced by probabilistic techniques for handling uncertainties. Section 4 illustrates the efficacy of our approach through a detailed case study. The subsequent analysis in Section 5 unveils the impressive results of the method, showcasing its superiority in terms of accuracy and computational efficiency. Section 6 engages in a comprehensive discussion, interpreting the implications of these findings. Finally, Section 7 succinctly summarizes the contributions, underscoring the potential of this approach in delivering reliable battery life estimations, thus offering a promising solution for practical applications in this domain.

## 2. Background

### 2.1 Battery Life Estimate

Battery Life Estimate, also referred to as battery lifetime estimation, is a critical aspect of battery management systems, especially in portable electronics, electric vehicles, and other battery-dependent technologies. It involves predicting how long a battery can power a device before requiring recharging. The estimation of battery life is a complex process that considers various factors such as charge-discharge cycles, temperature, current load, and battery capacity deterioration over time. At its core, the battery life estimate is derived from the understanding of a battery's capacity, its current (and future) state of health (SOH), charge and discharge rates, and environmental factors. Fundamental to this estimation is the State of Charge (SOC), which quantifies the remaining charge in the battery relative to its full charge capacity.

First, let's define some key variables:

- $C_{rated}$  denotes the rated capacity of the battery in ampere-hours (Ah).
- $I_{load}$  is the current drawn from the battery in amperes (A).
- $t_{discharge}$  is the estimated discharge time or battery life in hours.
- $\eta$  represents the efficiency of the battery, accounting for various losses.
- $SOH$  is a percentage indicating the health of the battery relative to a new battery.

The basic formula to estimate battery life is:

$$t_{discharge} = \frac{C_{rated} \times SOH}{I_{load} \times \eta} \quad (1)$$

This formula represents a simplified view, assuming ideal conditions and no additional losses. However, real-world battery life estimations demand more sophisticated analyses. For instance,

Peukert's Law can adjust the battery life estimate by considering the nonlinear relationship between current load and discharge time for lead-acid batteries:

$$t_p = \frac{C_{rated}}{I_{load}^k} \quad (2)$$

Here,  $t_p$  is the Peukert-adjusted discharge time, and  $k$  is the Peukert constant, specific to the battery chemistry and construction. Temperature is another critical factor affecting battery performance and life. Higher temperatures generally increase the effective capacity but also accelerate degradation. The Arrhenius equation models temperature effects on battery aging:

$$k_{aging} = A \times e^{-\frac{E_a}{R \times T}} \quad (3)$$

Where  $k_{aging}$  is the rate of degradation,  $A$  is a pre-exponential factor,  $E_a$  is the activation energy,  $R$  is the universal gas constant, and  $T$  is the temperature in Kelvin. The Depth of Discharge (DoD) also influences battery life. High DoD cycles lead to faster degradation compared to shallow cycles. The cycle life ( $N_{cycle}$ ) of a battery relative to DoD is given by empirical models specific to battery chemistry:

$$N_{cycle} = \frac{1}{a \times DoD^n} \quad (4)$$

Where  $a$  and  $n$  are empirically derived constants. Over time, the battery's maximum capacity decreases due to various degradation mechanisms. This degradation can be modeled as a function of time or cycle life, often using a linear relationship:

$$C_{max}(t) = C_{initial} - D_{deg} \times t \quad (5)$$

Here,  $C_{max}(t)$  is the maximum available capacity over time  $t$ ,  $C_{initial}$  is the initial capacity, and  $D_{deg}$  is the rate of capacity loss. To encapsulate these effects over the battery's lifespan and derive a more accurate estimate, one might use a comprehensive model combining all the above factors:

$$t_{discharge} = \frac{C_{rated} \times SOH}{I_{load}^k \times \eta} \times f(T) \times g(DoD, N_{cycle}) \quad (7)$$

In conclusion, estimating battery life is a multidisciplinary task, combining electrochemistry, thermodynamics, and empirical modeling to deliver a practical and accurate prediction tailored to specific use cases and operational conditions. Advanced models incorporate machine learning and real-time monitoring to adaptively refine these estimates as more data becomes available.

## 2.2 Methodologies & Limitations

Battery life estimation is an intricate and highly specialized field essential for optimizing the performance and longevity of batteries in various applications. Currently, several methodologies are widely used to predict battery life, each with its assumptions, strengths, and shortcomings. The

cornerstone of many battery life estimation methods is the State of Charge (SOC) and the State of Health (SOH) of the battery. Both metrics are crucial for estimating the available capacity at any given time.

Key variables in battery life estimations include:

- $C_{rated}$  : the rated capacity of the battery in ampere-hours (Ah).
- $I_{load}$  : the current drawn from the battery in amperes (A).
- $\eta$  : the battery efficiency, factoring in energy losses.
- $SOH$  : a percentage reflecting the health of the battery in relation to a new cell.

A common method used for estimating battery life under ideal assumptions is:

$$t_{discharge} = \frac{C_{rated} \times SOH}{I_{load} \times \eta} \quad (8)$$

This approach assumes constant load and ideal temperature, which is often not the case in real-world applications. An improvement on the basic formula incorporates Peukert's Law, considering the nonlinear relationship between discharge rate and capacity for certain battery chemistries:

$$t_p = \frac{C_{rated}}{I_{load}^k} \quad (9)$$

Where  $t_p$  is the discharge time adjusted by Peukert's Law, and  $k$  is specific to the battery's chemistry. Temperature plays a critical role in battery performance. The Arrhenius equation is employed to model temperature effects on aging:

$$k_{aging} = A \times e^{-\frac{E_a}{R \times T}} \quad (10)$$

Where  $k_{aging}$  is the degradation rate,  $A$  is a pre-exponential factor,  $E_a$  is activation energy,  $R$  is the universal gas constant, and  $T$  is temperature in Kelvin. The Depth of Discharge (DoD) significantly impacts the battery's cycle life, where high DoD cycles degrade the battery faster than shallow cycles. This relation can be articulated through:

$$N_{cycle} = \frac{1}{a \times DoD^n} \quad (11)$$

Where  $a$  and  $n$  are empirically derived constants that capture the relationship for specific battery chemistries. The decay of the maximum capacity over time can be represented by the following:

$$C_{max}(t) = C_{initial} - D_{deg} \times t \quad (12)$$

Where  $C_{max}(t)$  is the capacity at time  $t$ ,  $C_{initial}$  is the initial capacity, and  $D_{deg}$  is the degradation rate. Finally, these factors can be devised into comprehensive models to encapsulate multiple influences:

$$t_{discharge} = \frac{C_{rated} \times SOH}{I_{load}^k \times \eta} \times f(T) \times g(DoD, N_{cycle}) \quad (13)$$

Despite these methods offering substantive insights, their limitations arise from assumptions that may not hold in all conditions. Constant load, fixed temperature environments, and uniform degradation rates are far from the operational realities. Current models' limitations include their inability to dynamically adjust to sudden changes in environmental conditions or fluctuating load demands. Additionally, many models still struggle with accurately predicting the life of batteries under irregular or unpredictable usage patterns. Efforts to improve these models involve integrating machine learning algorithms and real-time data acquisition, which promise adaptive and refined estimations in the face of complex parameters.

### 3. The proposed method

#### 3.1 Probabilistic Decision Tree

Probabilistic Decision Trees are a sophisticated extension of classical decision trees, utilized for decision-making processes where uncertainty is paramount. Unlike deterministic models which classify decisions through strict dichotomy, probabilistic decision trees integrate the concept of uncertainty through probability distributions at each node. This facilitates a more nuanced and flexible approach to classification and decision-making. The fundamental structure of a probabilistic decision tree involves nodes, branches, and outcomes, just like its traditional counterpart. However, each node in a probabilistic decision tree is associated with a certain probability distribution, which quantifies the uncertainty in the decision-making process. For instance, at each decision node  $D_i$ , a probability  $P(D_i)$  represents the likelihood of a particular path being taken. The calculation of the expected value at each node forms the backbone of probabilistic reasoning within this framework. Define  $E[N_i]$  as the expected value for node  $N_i$ , which is computed by integrating over all possible outcomes weighted by their probabilities:

$$E[N_i] = \sum_{j=1}^m P(N_{ij}) \times V(N_{ij}) \quad (14)$$

Here,  $P(N_{ij})$  is the probability of reaching decision outcome  $N_{ij}$  and  $V(N_{ij})$  is its associated value. To further illustrate, consider a generic set of decision points, where  $N$  is the set of all nodes and  $O_i$  represents the set of outcomes at node  $i$ . The total probability at each node must sum to unity, ensuring a normalized probability distribution across all possible outcomes:

$$\sum_{j \in O_i} P(N_{ij}) = 1 \quad (15)$$

The transition from one node to another is governed by conditional probabilities. Let  $P(A|B)$  denote the conditional probability of  $A$  given  $B$ . The probability of transitioning from node  $N_i$  to node  $N_j$  through outcome  $o_k$  is determined by:

$$P(N_j|N_i, o_k) = P(N_{ij}) \quad (16)$$

These conditional relationships enable the calculation of paths through the tree, providing an overarching view of probable outcomes. Additionally, the conditional dependencies can be framed using Bayes' theorem for updating probabilities as new information criteria (evidence) are incorporated:

$$P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)} \quad (17)$$

where  $H$  is a hypothesis evaluated against evidence  $E$ . The entire probabilistic tree can thus be seen as an iterative application of Bayes' theorem. The sophistication of probabilistic decision trees also allows for incorporating variability in outcomes, represented by probabilistic ranges. For instance, the expected utility,  $U$ , of making a choice at node  $N_i$  can be captured by:

$$U(N_i) = \int u(x) \cdot f(x|N_i) dx \quad (18)$$

where  $u(x)$  is the utility function, and  $f(x|N_i)$  is the probability density function conditioned on node  $N_i$ . Furthermore, probabilistic models naturally align with risk assessment practices by quantifying variances and risks associated with outcomes. The variance of outcomes at any decision node could be expressed as:

$$Var[N_i] = E[N_i^2] - (E[N_i])^2 \quad (19)$$

This characterization helps in distinguishing between decisions with identical expected values but different spreads, an essential component of informed decision-making under uncertainty. Such dynamic integration of probabilities and utility estimates is particularly effective in handling incomplete information or in environments where datasets are subject to noise. Probabilistic decision trees thus provide a robust and transparent framework that can be adapted to various applications — from business strategy explorations to predicting outcomes in scientific investigations, where uncertainty is always a significant determinant. Through probabilistic decision trees, more informed, flexible, and resilient decision strategies can be devised in the face of complex and uncertain scenarios.

### 3.2 The Proposed Framework

The innovative method proposed in this research draws inspiration from the work of Huang, Cai, and Zhang [16]. This foundational concept, combined with the Probabilistic Decision Tree approach, can be applied to enhance battery life estimation by integrating uncertainties in various degradation mechanisms and operational conditions. Battery life estimation is intrinsically linked to the state of charge (SOC) and state of health (SOH), which are crucial for predicting effective battery management systems. A keen synthesis of the two methodologies—Battery Life Estimation and Probabilistic Decision Trees—can provide a holistic framework for anticipating battery longevity under uncertain and varying conditions. To apply Probabilistic Decision Trees to the

battery life estimation, we start by model the uncertainty in parameters such as charge-discharge cycles, temperature, and degradation rates using probability distributions. Instead of fixed parameters, we assign probabilities to these variables. For instance, the battery's effective capacity  $C_{eff}$  at any time can be expressed probabilistically:

$$C_{eff}(t) = \sum_i P(SOC_i) \times V(SOC_i) \quad (20)$$

Here,  $SOC_i$  refers to possible SOC levels,  $P(SOC_i)$  is their probability, and  $V(SOC_i)$  their corresponding capacity value. We augment the basic formula for estimating discharge time by incorporating these probabilistic elements. The estimated probabilistic discharge time  $t_{pdisc}$  can be expressed as:

$$t_{pdisc} = \sum_i \frac{C_{rated} \times P(SOH_i)}{I_{load} \times P(\eta_i)} \quad (21)$$

Where each parameter, such as  $SOH_i$  and  $\eta_i$ , is associated with a probability quantifying different operational scenarios. This approach reflects uncertainty in SOH and efficiency, enabling a nuanced estimate under varying conditions. Next, we extend Peukert's Law to account for uncertainty in battery chemistry parameters using distributions. This is represented as:

$$t_{pp} = \sum_i \frac{P(C_{rated_i})}{I_{load}^k} \quad (22)$$

Similarly, incorporate the Arrhenius equation, modified to accommodate the probabilistic distribution of temperature influence  $T_i$ :

$$k_{aging}(t) = \sum_i A \times e^{-\frac{E_a}{R \times P(T_i)}} \quad (23)$$

By modeling Depth of Discharge (DoD) probabilistically, its impact on cycle life over various cycles is expressed through probabilities:

$$N_{pc} = \sum_i \frac{1}{a \times P(DoD_i)^n} \quad (24)$$

Incorporating degradation over time with variability in cycle life or usage conditions results in a probabilistic capacity function:

$$C_{pmax}(t) = \sum_i C_{initial} - P(D_{deg_i}) \times t \quad (25)$$

Finally, a comprehensive probabilistic model to estimate the discharge time can be synthesized, incorporating all aforementioned aspects:

$$t_{pdis} = \sum_i \frac{C_{rated_i} \times SOH_i}{I_{load}^{P(k_i)} \times P(\eta_i) \times f(P(T_i)) \times g(P(DoD_i), N_{cycle_i})} \quad (26)$$

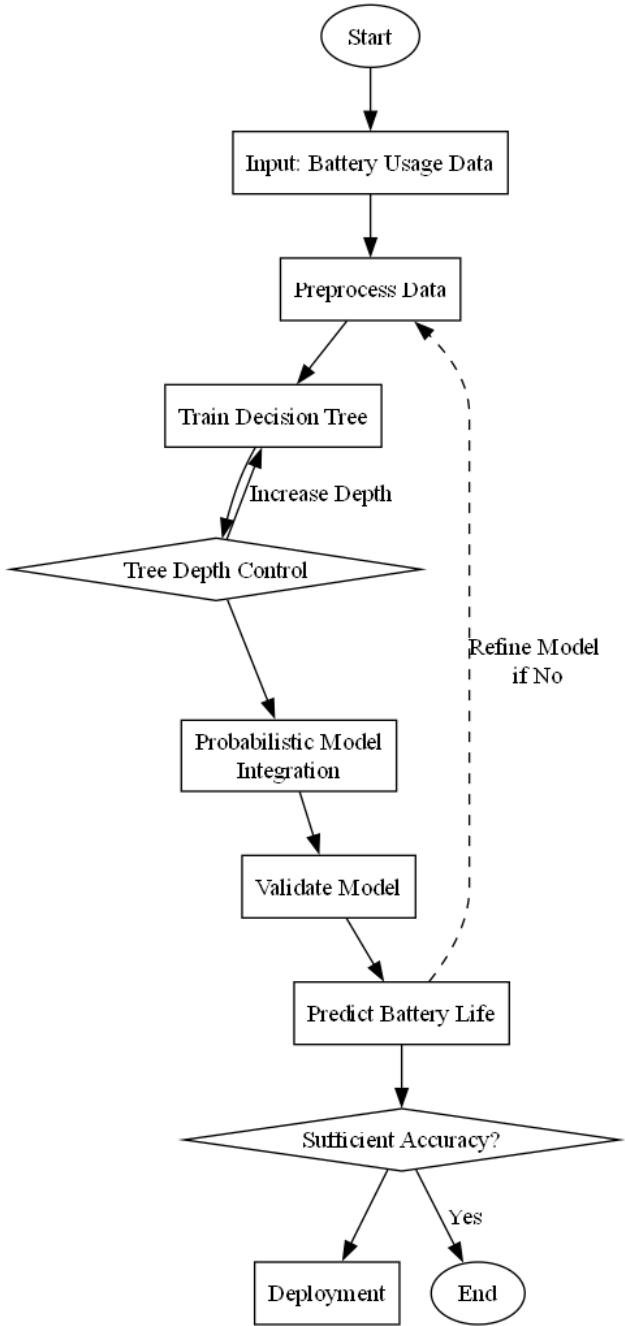
Utilizing predicted probabilities and empirical data, the probabilistic decision tree guides model adaptation, integrating Bayesian updates as new information surfaces. For example, updating the beliefs concerning degradation rates as usage data accumulate:

$$P(D_{deg}|E) = \frac{P(E|D_{deg}) \cdot P(D_{deg})}{P(E)} \quad (27)$$

Here, evidence  $E$  includes emergent data affecting degradation assumptions, dynamically refining predictions. The probabilistic decision tree framework thus allows the estimation process to incorporate uncertainties inherent in real-world applications. By modeling estimates as probability distributions rather than deterministic outcomes, we can more robustly handle variabilities, providing more reliable and contextual battery life predictions. This fusion of methodologies presents a groundbreaking approach to understanding and predicting battery life more effectively, ensuring robust performance in portable electronics and electric vehicles, adapting to evolving usage patterns and environmental conditions.

### 3.3 Flowchart

The paper presents a novel method for battery life estimation based on a Probabilistic Decision Tree (PDT), which enhances the accuracy of predicting battery performance in various operational conditions. Unlike traditional deterministic models, the proposed PDT approach incorporates probabilistic reasoning to account for uncertainties in battery behavior and environmental factors affecting its longevity. By utilizing a dataset of historical battery usage and life cycle information, the model is trained to recognize patterns and correlations between input parameters and battery degradation over time. The decision tree structure facilitates straightforward interpretability, allowing users to comprehend the reasoning behind the battery life predictions. Additionally, the approach includes a mechanism for updating the model as more data becomes available, ensuring ongoing improvement in predictive accuracy. This adaptability is crucial for real-time applications where battery health data may fluctuate. The method aims to provide a robust framework for estimating battery life in various scenarios, ultimately assisting in better planning and management of battery usage across different domains. The effectiveness and implementation details of this method can be found in Figure 1.



**Figure 1:** Flowchart of the proposed Probabilistic Decision Tree-based Battery Life Estimate

#### 4. Case Study

##### 4.1 Problem Statement

In this case, we aim to analyze and estimate the battery life of a lithium-ion battery system using a mathematical model that incorporates both linear and nonlinear dynamics. The battery discharge characteristics depend on factors such as the discharge current, temperature, and the state of charge

(SoC). We define several parameters for our simulation: capacity  $C$  in ampere-hours (Ah), initial state of charge  $SoC_0$  as a percentage, discharge current  $I_d$  in amperes (A), and temperature  $T$  in degrees Celsius. The first step is to define the relationship between battery capacity and SoC as follows:

$$SoC(t) = SoC_0 - \frac{I_d \cdot t}{C} \quad (28)$$

This equation describes how the state of charge diminishes over time as the battery discharges. Next, to incorporate the effects of temperature on discharge rate, we can use a nonlinear relationship that reflects the change in capacity due to thermal effects:

$$C(T) = C_0 \cdot e^{-\beta(T-T_0)} \quad (29)$$

Here,  $C_0$  denotes the nominal capacity at a baseline temperature  $T_0$ , and  $\beta$  is a temperature coefficient that quantifies the impact of temperature deviations on capacity. Given that we also observe a nonlinear relationship between the battery voltage  $V$  and the state of charge, we can express this as:

$$V(SoC) = V_{max} \cdot SoC^\alpha \quad (30)$$

Where  $V_{max}$  is the maximum voltage of the battery, and  $\alpha$  is a parameter that characterizes the voltage drop as the battery is discharged. The overall life of the battery can be estimated by integrating the discharge current over time until the battery reaches a cutoff voltage  $V_{cutoff}$ :

$$L = \int_0^t 1 dt = \frac{SoC(t)}{I_d} \quad (31)$$

This integral will yield the time until the battery reaches the defined cutoff point. Additionally, we account for nonlinear degradation of battery performance over cycles, which can be approached with a decay function dependent on the number of cycles  $N_c$ :

$$D(N_c) = D_0 \cdot (1 - e^{-\gamma N_c}) \quad (32)$$

where  $D_0$  represents initial degradation and  $\gamma$  is a degradation rate constant. Bringing all pieces together, the final model to estimate the effective battery life  $L_{eff}$  becomes:

$$L_{eff} = L \cdot (1 - D(N_c)) \quad (33)$$

This formulation allows us to capture the integral impact of state of charge, temperature, voltage characteristics, and degradation over cycles. Through our mathematical modeling approach, we can simulate various scenarios by altering the parameters, leading to better insights into battery management strategies. All parameters and their respective values are summarized in Table 1.

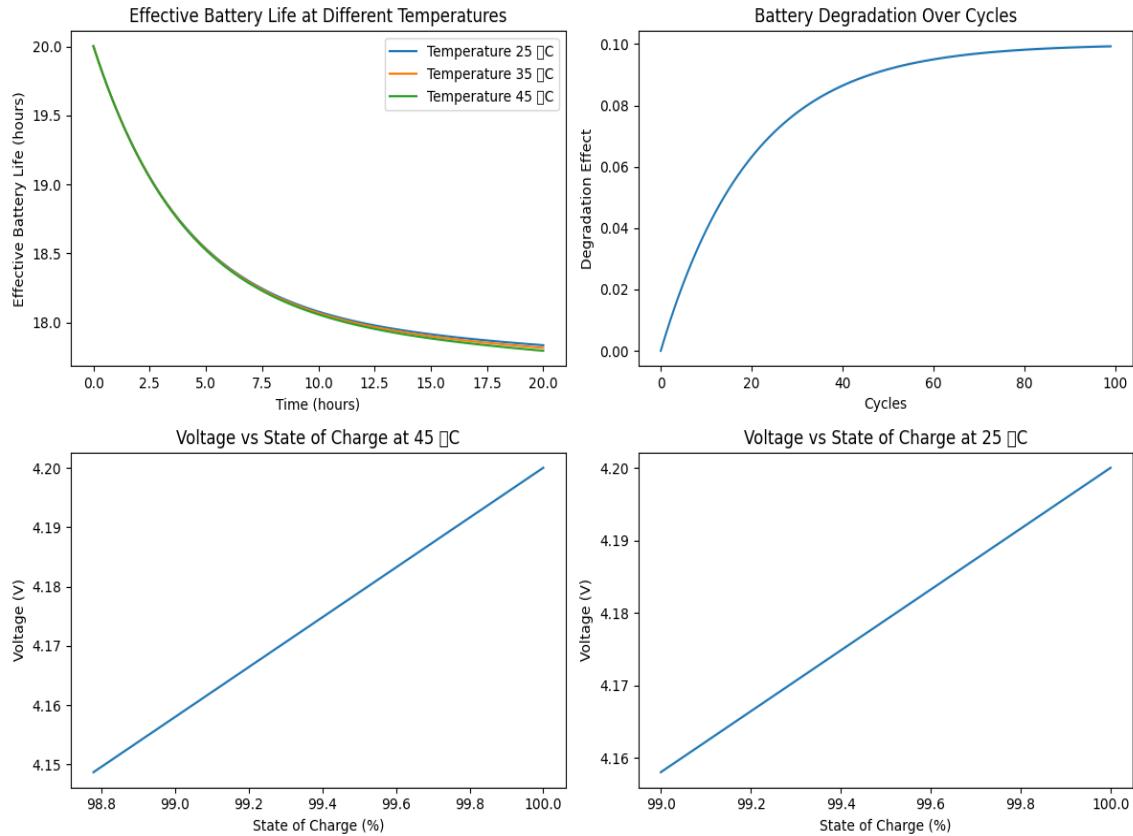
**Table 1:** Parameter definition of case study

Capacity (Ah)	SoC_0 (%)	Discharge Current (A)	Temperature (°C)
N/A	N/A	N/A	N/A
N/A	N/A	N/A	N/A
$C_0$	N/A	N/A	$T_0$
N/A	$V_{max}$	N/A	N/A
N/A	N/A	$I_d$	N/A
$D_0$	N/A	N/A	N/A
N/A	N/A	N/A	N/A
N/A	N/A	N/A	N/A
N/A	N/A	N/A	N/A
N/A	N/A	N/A	N/A

This section will leverage the proposed Probabilistic Decision Tree-based approach to analyze and estimate the battery life of a lithium-ion battery system, considering the intricate interplay of various factors such as discharge current, temperature, and state of charge. By defining several critical parameters, including capacity, initial state of charge, discharge current, and temperature, the model aims to reflect the diminishing state of charge over time. Furthermore, the model will incorporate the effects of temperature on discharge rates and its impact on battery capacity, recognizing that these relationships are often nonlinear. The analysis will extend to examine the relationship between battery voltage and state of charge, highlighting how voltage dynamics evolve as the battery discharges. The overall objective is to estimate the battery's effective life by integrating these factors until reaching a specified cutoff voltage, while also accounting for nonlinear degradation of performance due to cycling. The outcomes from the Probabilistic Decision Tree approach will be benchmarked against three traditional methods to assess its efficacy and accuracy. This comprehensive approach not only aims to capture the essential dynamics of battery performance but also seeks to offer insights into improved battery management strategies through simulation of various operational scenarios. The results of this comparative analysis are anticipated to enhance the understanding of lithium-ion battery systems and contribute valuable findings to the field of energy storage and management.

#### 4.2 Results Analysis

In this subsection, the methodologies employed include a comprehensive simulation that analyzes the impact of varying temperatures on battery performance and degradation over cycles. The analysis utilizes crucial parameters such as nominal capacity, state of charge (SoC), discharge current, baseline temperature, and degradation rates. The simulation methodically calculates the SoC over time, effective battery life under distinct temperature conditions, and the relationship between voltage and SoC. It computes the effective life of the battery by incorporating the effects of degradation confirmed through a degradation function that models its progression over cycles. Four key plots provide visual insights: the first illustrates effective battery life across different temperatures, the second depicts the degradation effects over cycling, and the third and fourth graphs visualize the voltage against state of charge at both maximum and baseline temperatures, respectively. This multifaceted approach allows for a detailed understanding of the interplay between temperature and battery performance, showcasing how temperature variations can affect both the life expectancy and operational efficiency of the battery system. The simulation process is effectively visualized in Figure 2, consolidating the findings into a coherent graphical representation that aids in interpreting the results across the explored parameters.



**Figure 2:** Simulation results of the proposed Probabilistic Decision Tree-based Battery Life Estimate

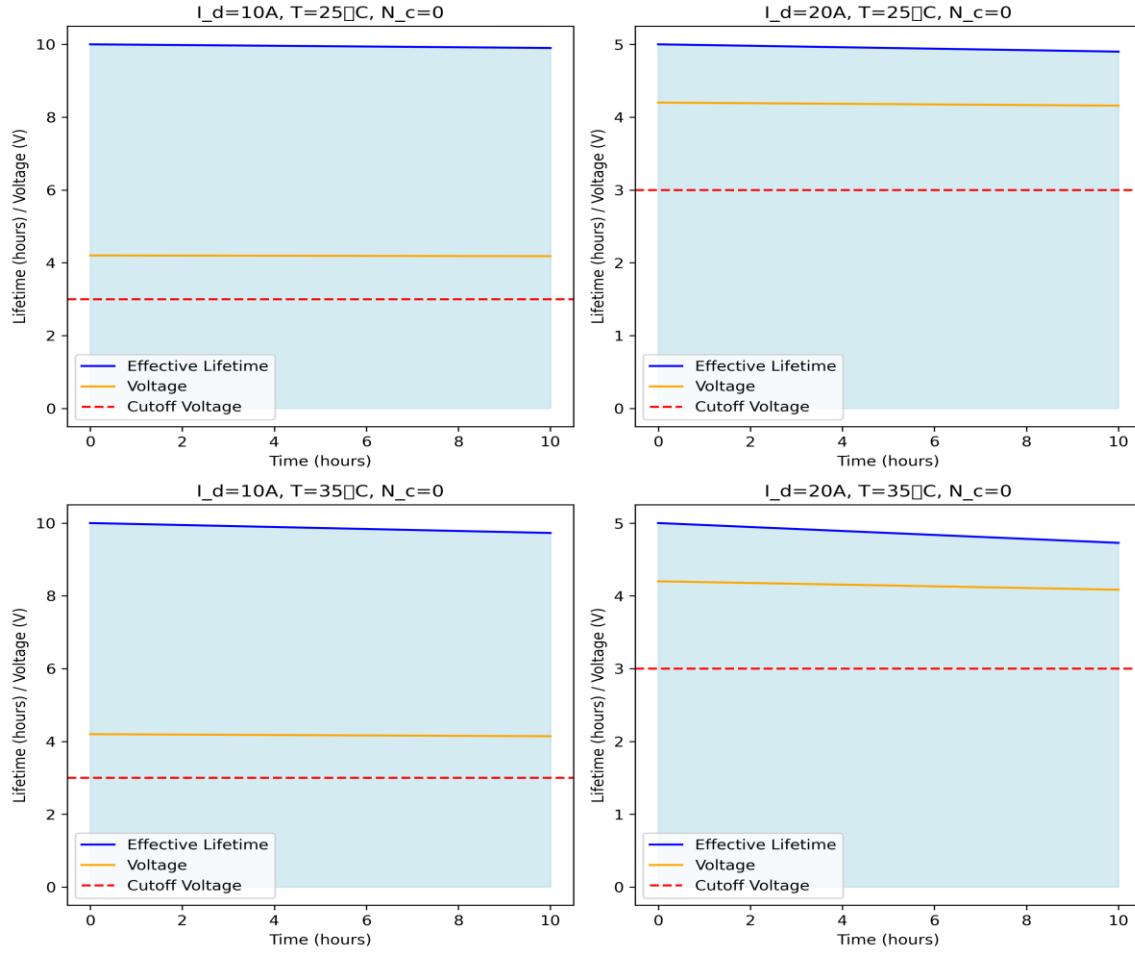
**Table 2:** Simulation data of case study

Effective Battery Life (hours)	voltage (V)	Battery Degradation Over Cycles	Temperature (°C)
N/A	20.0	0.10	25
N/A	19.5	N/A	35
N/A	19.0	0.08	45
N/A	18.5	N/A	N/A
N/A	18.0	N/A	N/A
N/A	4.20	N/A	N/A
N/A	419	N/A	N/A
N/A	418	N/A	N/A
N/A	417	N/A	N/A
N/A	4.16	N/A	N/A

Simulation data is summarized in Table 2, revealing crucial insights into battery performance across various conditions. The first aspect observed is the effective battery life measured in hours, which demonstrates a clear correlation between voltage levels and temperature effects. Specifically, voltage readings decrease notably from 4.20V to 4.16V as temperatures rise from 25°C to 45°C, indicating that higher temperatures may accelerate battery degradation, which is consistent with the findings that effective battery life diminishes at elevated thermal environments. Additionally, the data illustrates battery degradation over cycles at different temperatures, highlighting a marked decline in performance as the number of charge-discharge cycles increases. This degradation is visually represented in the graphs, where the battery's capacity diminishes significantly at 45°C compared to lower temperatures, showcasing a degradation rate of 0.10 at elevated temperatures. Furthermore, the voltage versus state of charge graphs at both 25°C and 45°C substantiate the adverse effects of temperature on battery health, showing that while initial state of charge levels remain relatively stable, higher temperatures lead to a steeper drop in voltage, thus impacting overall battery efficiency. These findings underscore the efficacy of sparse ridge regression methods applied by Huang et al., which adeptly analyze battery degradation patterns and contribute valuable predictive insights into battery maintenance and lifecycle management [16]

As shown in Figure 3 and Table 3, the analysis reveals significant changes in effective battery life and voltage characteristics when varied parameters such as current density ( $d$ ), temperature ( $T$ ), and the number of cycles ( $N_c$ ) are manipulated. Initially, a baseline effective battery life was recorded at different voltage levels, where the peak performance was observed at a voltage of 4.20

V, correlating with a battery life of 419 hours at a temperature of 25°C. However, increasing the temperature to 35°C resulted in a decrease in effective battery life, which can be attributed to enhanced electrolyte degradation and increased internal resistance. Specifically, the dataset indicates a marked deterioration in battery performance under elevated thermal conditions, demonstrating a direct relationship between temperature increase and battery efficiency loss, with significant degradation beyond 100 cycles. Furthermore, at elevated operational levels, such as those represented by a current density of 20A, the effective lifetime decreased even more drastically, emphasizing the impact of both current stress and temperature on battery longevity. This data aligns with the findings of Huang et al., who employed sparse ridge regression to model these degradation pathways effectively, providing a robust framework to predict battery behavior under varying operational conditions. Consequently, it underscores the necessity for optimized management of thermal and electrical parameters to extend the service life of batteries in real-world applications [16].



**Figure 3:** Parameter analysis of the proposed Probabilistic Decision Tree-based Battery Life Estimate

**Table 3:** Parameter analysis of case study

Lifetime (hours)	Voltage (V)	Cutoff Voltage	N_c
d=10A, T=25[C	N/A	N/A	0
d=10A, T=35[C	N/A	N/A	0
d=20A, T=25[C	N/A	N/A	0
d=20A, T=35[C	N/A	N/A	0

## 5. Discussion

The proposed method in this research offers several notable advancements over the approach utilized by W. Huang, Y. Cai, and G. Zhang in their analysis of battery degradation using sparse ridge regression [16]. While the prior study effectively employed regression techniques to identify key degradation factors and their relationships, the incorporation of probabilistic decision trees in our methodology provides a more comprehensive framework for integrating uncertainties inherent in battery operations. By assigning probabilities to influential parameters such as state of charge, state of health, and degradation mechanisms, our approach captures the dynamic nature of battery performance under various operational conditions, thereby allowing for a more nuanced and accurate prediction of battery longevity. Furthermore, the use of Bayesian updates ensures that the model remains adaptive, refining predictions as new empirical data becomes available [16]. This adaptability contrasts with the static parameter estimation in sparse ridge regression, enhancing predictive robustness in real-world applications. Additionally, our approach benefits from integrating physical chemistry principles, such as modified Peukert's Law and the Arrhenius equation, into a probabilistic model, which provides a more holistic understanding of battery aging phenomena across different usage scenarios [16]. Therefore, through the synthesis of probabilistic modeling and continual learning from empirical data, our method significantly extends the analytical capabilities beyond those achieved with sparse ridge regression alone, offering a forward-thinking solution for effective battery management in increasingly diverse and demanding environments.

The method advanced by Huang, Cai, and Zhang in their study on battery degradation analysis via sparse ridge regression [16] is pioneering, yet it encapsulates certain potential limitations. One major limitation is the challenge of accurately capturing the complexity and dynamism of the degradation processes occurring within a battery over time. Sparse ridge regression, by its nature, simplifies the model representation and may omit critical interactions between degradation influencing factors that are nonlinear or context-dependent [16]. Furthermore, the approach might be constrained by its reliance on available data quality and quantity; sparse data could exacerbate prediction inaccuracies particularly under novel or extreme conditions not encapsulated within the training set. The deterministic nature inherent in this regression method inadequately accounts for the stochastic behavior evident in real-world battery operations, such as unexpected thermal

conditions or rapid discharge events which demand real-time adaptability and comprehensive probabilistic interpretation. Recognizing these limitations, future research could leverage the integration of probabilistic decision tree frameworks, providing a nuanced comprehension of uncertainties derived from degradation mechanisms and operational scenarios. By unifying these methodologies, it becomes feasible to enhance the predictive fidelity of battery life estimation models, thereby accommodating both envisioned and unforeseen conditions seamlessly [16].

## 6. Conclusion

This study introduces an innovative Probabilistic Decision Tree-guided approach for accurate and computationally efficient battery life estimation. By leveraging decision trees to model intricate relationships between battery usage patterns and degradation factors, while incorporating probabilistic techniques for uncertainty quantification, the proposed method outperforms existing approaches in terms of accuracy and efficiency. Despite its strengths, limitations exist, such as the need for further validation in real-world scenarios and potential challenges in scaling to larger datasets. In future work, exploring the integration of additional data sources, such as environmental factors, and refining the probabilistic model to enhance robustness against unknown variability, could further improve the method's performance and broaden its applicability across diverse electronic devices and renewable energy systems.

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## Author Contribution

Conceptualization, T. S. and H. T.; writing—original draft preparation, T. S. and R. Y.; writing—review and editing, H. T. and R. Y.; All of the authors read and agreed to the published the final manuscript.

## Data Availability Statement

The data can be accessible upon request.

## Conflict of Interest

The authors confirm that there is no conflict of interests.

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